2.5 Macroscopic Quantum State

At what velocity vortices are formed ?

critical angular velocity

$$\omega_c = E_v / L_v$$

$$L_v = \int_0^R \varrho_s r\, v_s\, 2\pi r\, dr = \frac{1}{2} \varrho_s \kappa R^2$$

angular momentum

$$\Rightarrow \quad \omega_c = \frac{h}{2\pi m_4 R^2} \ln\left(\frac{R}{a_0}\right)$$

$$R = 1\,\mathrm{cm} \longrightarrow \omega_c \approx 10^{-3}\,\mathrm{s}^{-1}$$

comment:

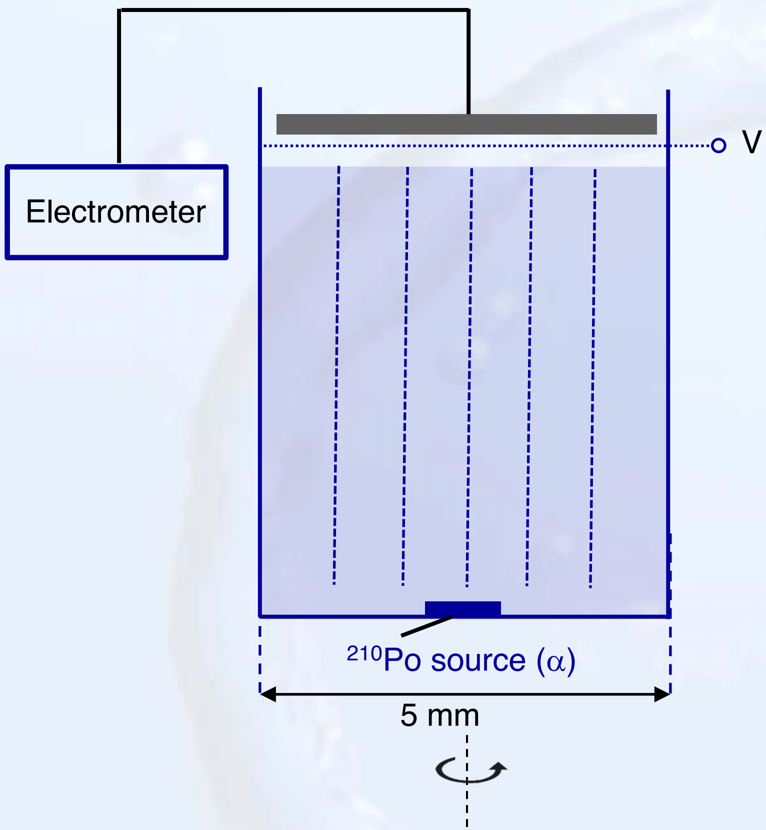
concept of critical velocity will be discussed in section 2.6

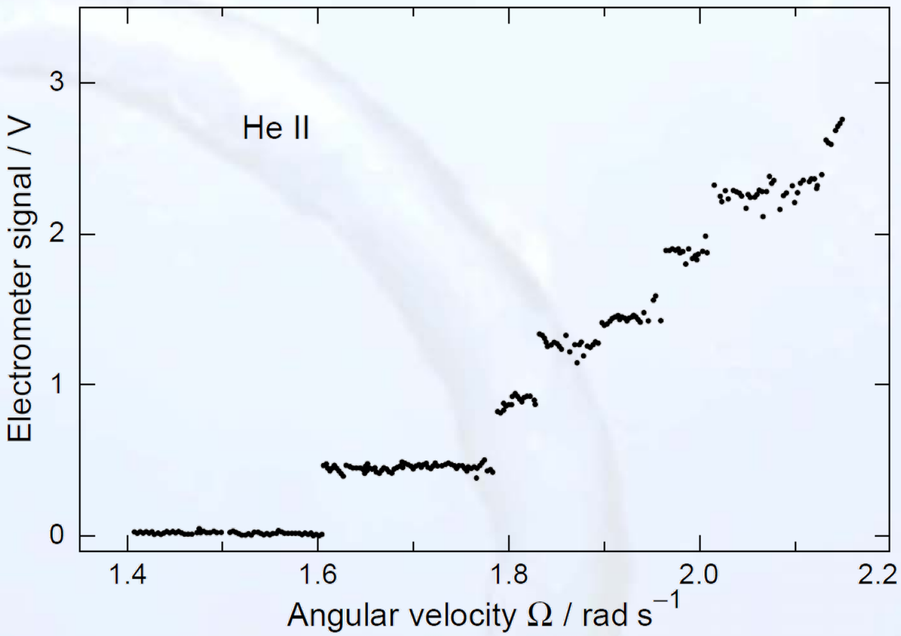
Experimental observation of vortices

- meniscus is rotating vessels
- damping of second sound
- electrometer experiments
- exploding electron bubbles
- decorating with hydrogen ice particles

2.5 Macroscopic Quantum State

Experimental observation of vortices

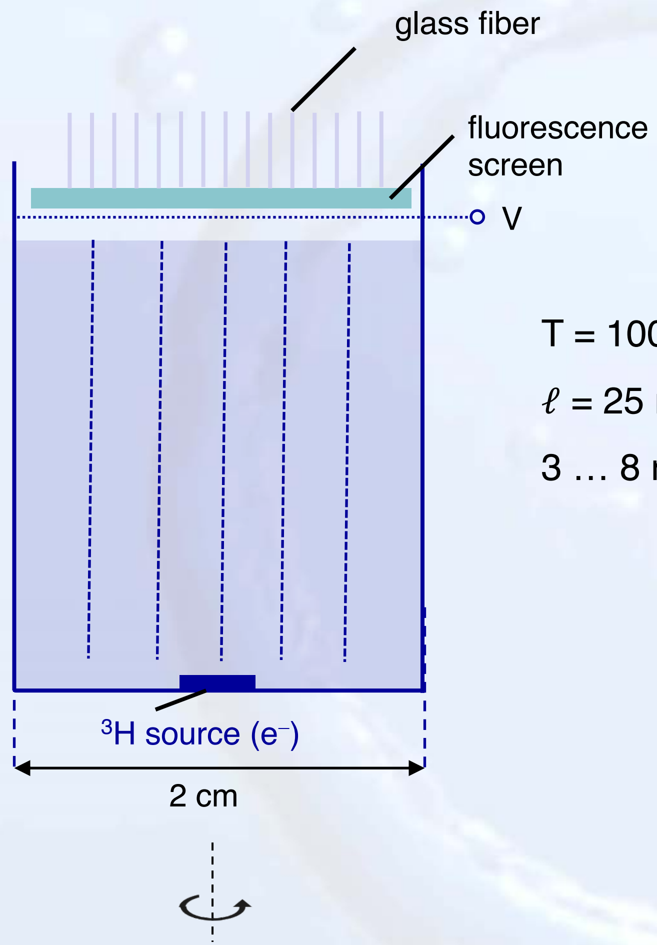




- α source → helium ionized → electrons form bubbles
- bubbles are captured by vortex lines via Magnus force
- *E* field is pulling bubbles alongside of vortex line to surface
- measurement of charge → is proportional to number vortex lines
- uniform acceleration over 10 h to 10 rot/min

2.5 Macroscopic Quantum State

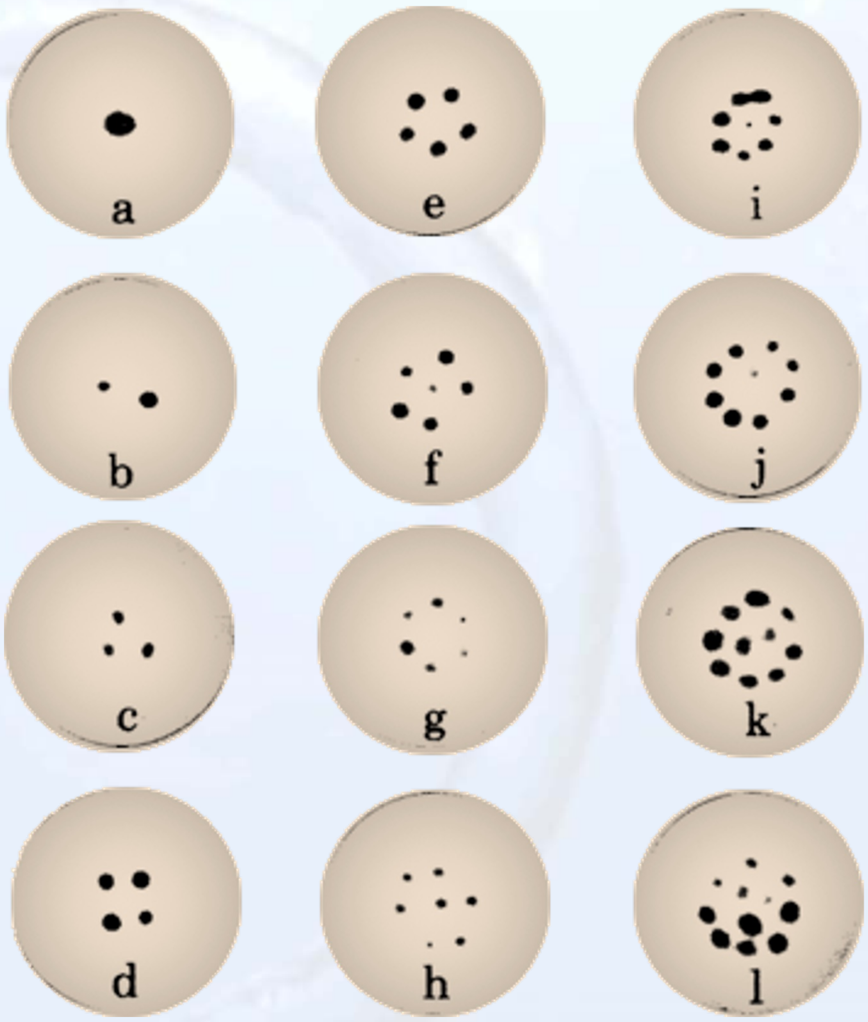
Experimental observation of vortices



T = 100 mK

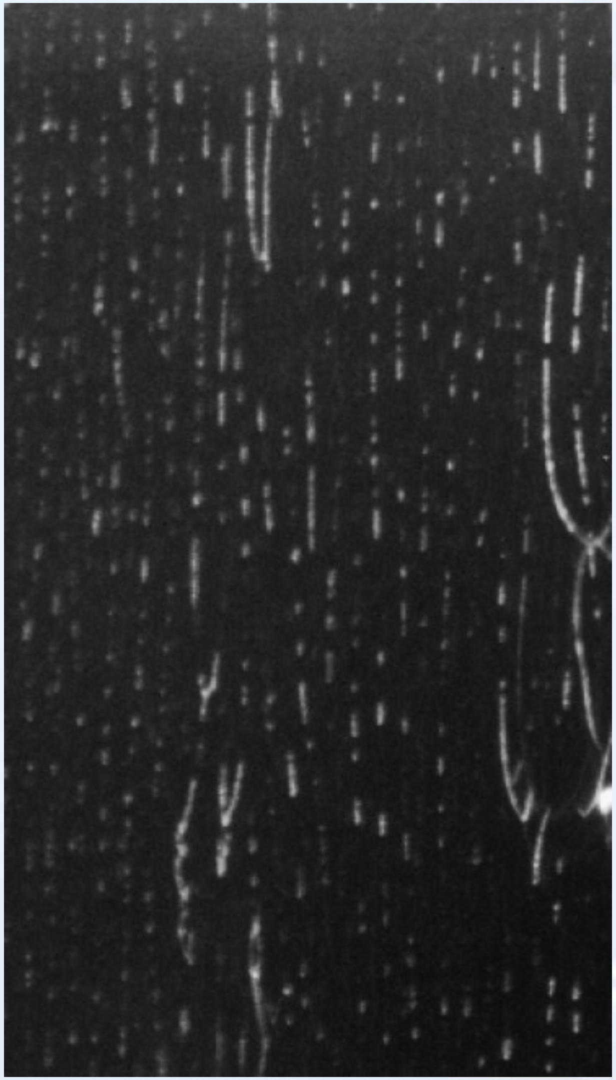
ℓ = 25 mm

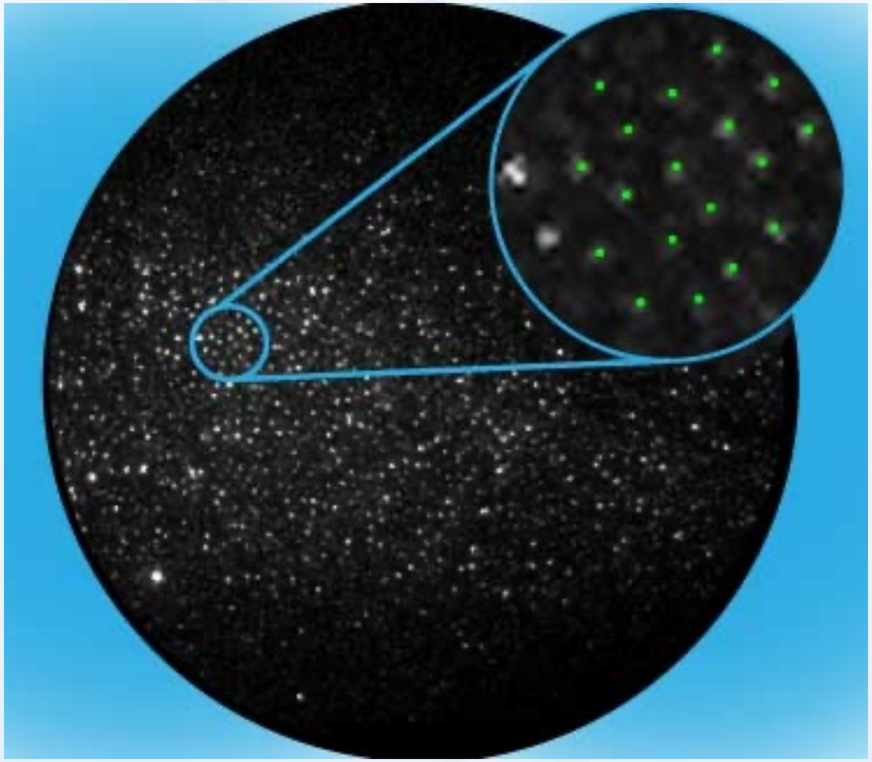
3 … 8 rot/min



2.5 Macroscopic Quantum State

Experimental observation of vortices

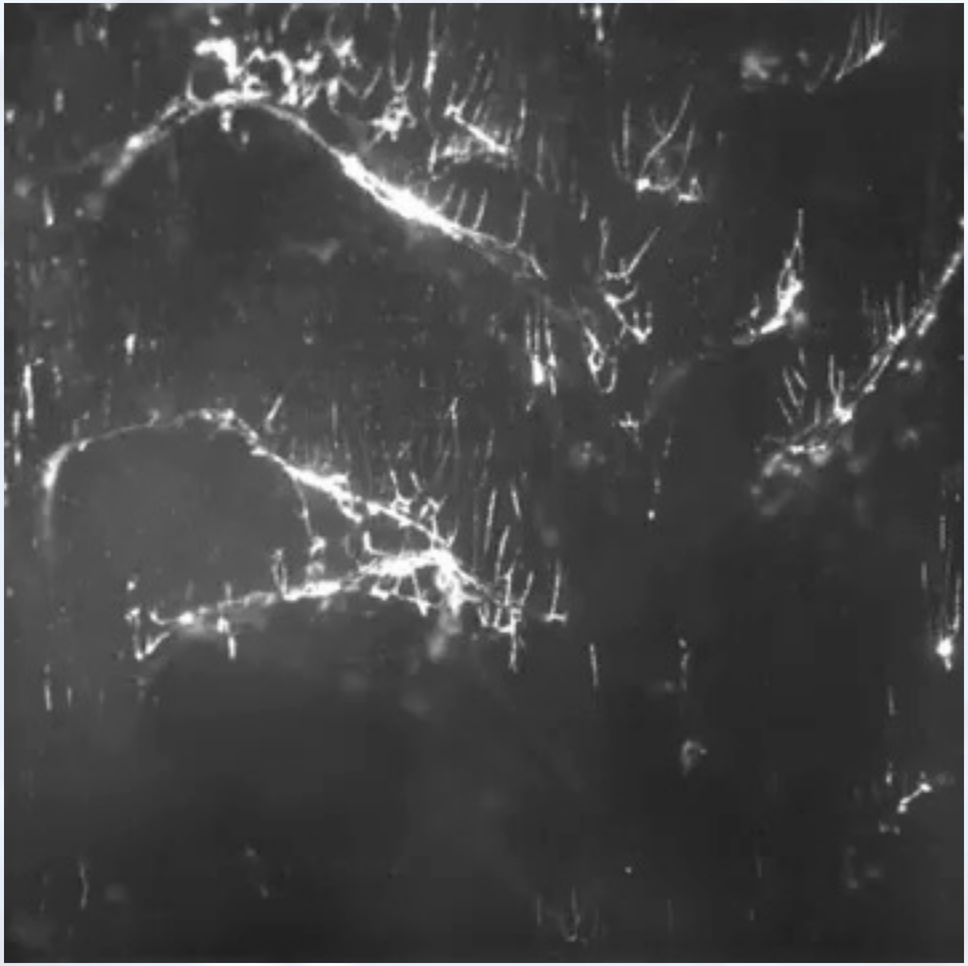




Abrikosov lattice ⟶ Type 2 superconductor

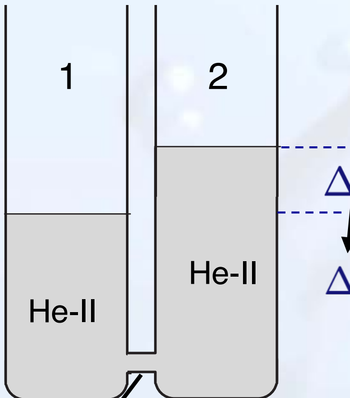
2.5 Macroscopic Quantum State

Experimental observation of vortices



2.5 Macroscopic Quantum State

Josephson Effects



Δz

$$\Delta\mu = \mu_2 - \mu_1 = m_4 g \Delta z$$

weak link

healing length

$$d \approx \xi = 1 \ldots 2\,\text{Å}$$

$$\xi_4 = \frac{0.3\,\text{nm}}{(1 - T/T_\lambda)^{2/3}}$$

diverges for $T \to T_\lambda$

Schrödinger Eq.

$$\mathrm{i}\hbar\dot{\Psi}_1 = \mu_1\Psi_1 + \mathcal{K}\Psi_2$$

$$\mathrm{i}\hbar\dot{\Psi}_2 = \mu_2\Psi_2 + \mathcal{K}\Psi_1$$

with $\Psi_1 = \sqrt{\varrho_\mathrm{s}}\mathrm{e}^{\mathrm{i}\varphi_1}$ and $\Psi_2 = \sqrt{\varrho_\mathrm{s}}\mathrm{e}^{\mathrm{i}\varphi_2}$

$$\frac{\partial \varrho_\mathrm{s}}{\partial t} = \frac{2\mathcal{K}}{\hbar}\,\varrho_\mathrm{s}\sin(\varphi_2 - \varphi_1)$$

$T = 0$

$$\frac{\partial}{\partial t}(\varphi_2 - \varphi_1) = -\frac{1}{\hbar}(\mu_2 - \mu_1) = -\frac{1}{\hbar} m_4 g \Delta z$$

$\Delta\mu = 0$ phase difference constant → Josephson dc effect

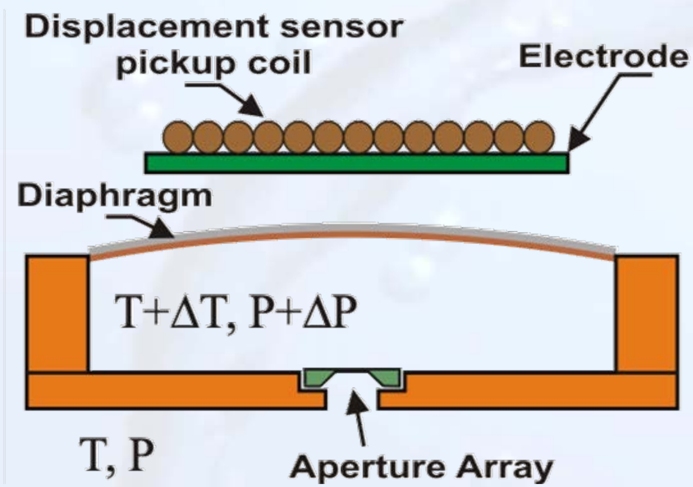
$\Delta\mu \neq 0$ phase difference changes → Josephson ac effect

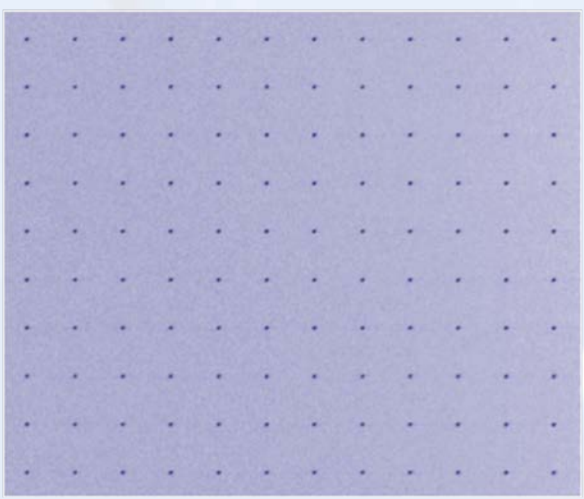
$$T = 0 \longrightarrow \omega_\mathrm{J} = \frac{\Delta\mu}{\hbar} = \frac{m_4 \Delta p}{\hbar\varrho}$$ with $\Delta p = \varrho g \Delta z$

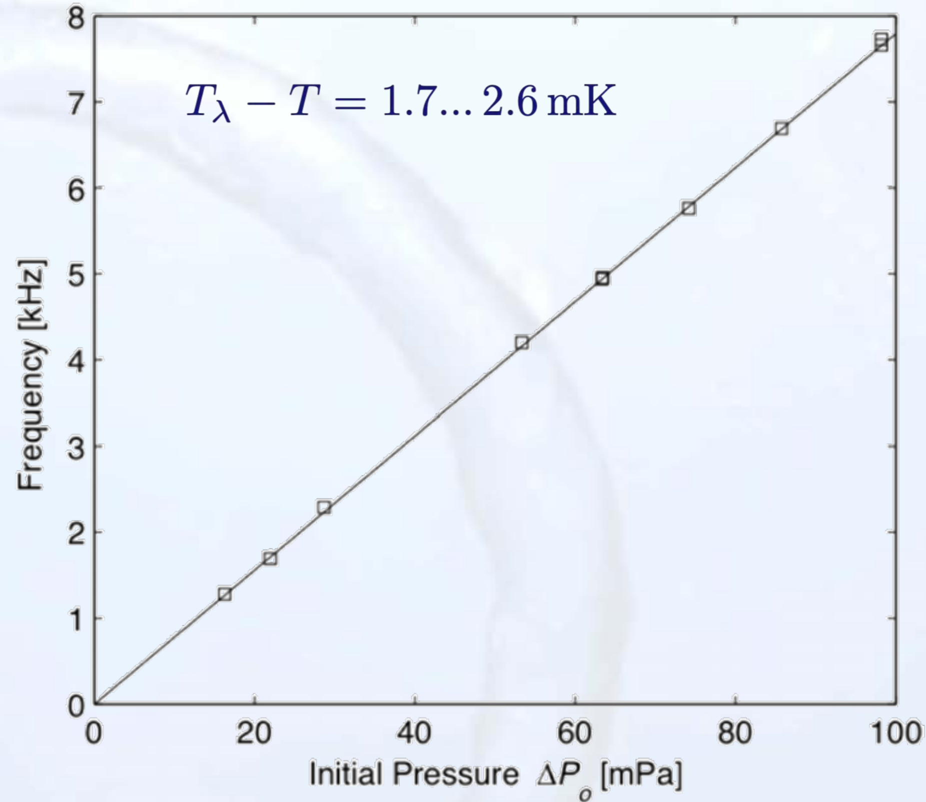
$$T \neq 0 \longrightarrow \omega_\mathrm{J} = \frac{\Delta\mu}{\hbar} = \frac{m_4 \Delta p}{\hbar\varrho} - m_4\frac{S\Delta T}{\hbar}$$

2.5 Macroscopic Quantum State

Josephson Effects







measurement directly after pressure applied

$\longrightarrow \Delta T \approx 0$

65×65 array

$70\,\mathrm{nm}$ apertures spaced $3\,\mu\mathrm{m}$

50 nm-thick silicon nitride membrane

2.6 Excitation Spectrum of He-II: Landau Model

Is the occurrence of the condensate equivalent to superfluidity ?

comment:

superconductivity in metals is related to an energy gap

nature of excitations is important

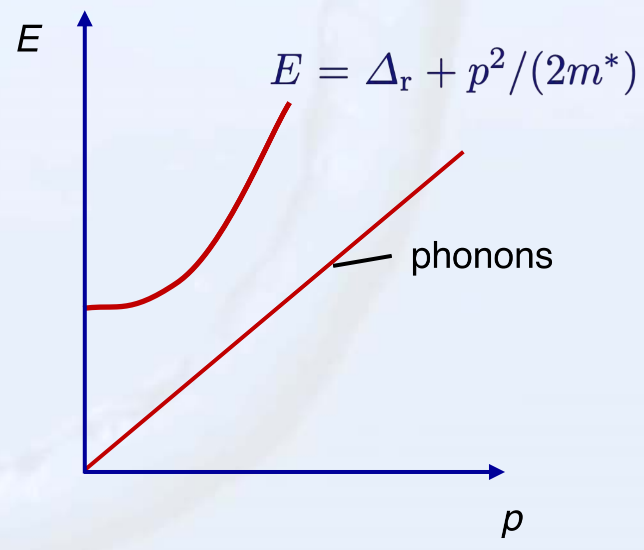
ideal Bose gas:

$$E = \frac{p^2}{2m} \longrightarrow$$ arbitrary energy transfer possible

➡ no superfluidity **!**

Idea of Landau 1941

- at low temperatures: only longitudinal phonons with linear dispersion
- at "high" temperatures: more and other kinds of excitations contribute, but with energy gap



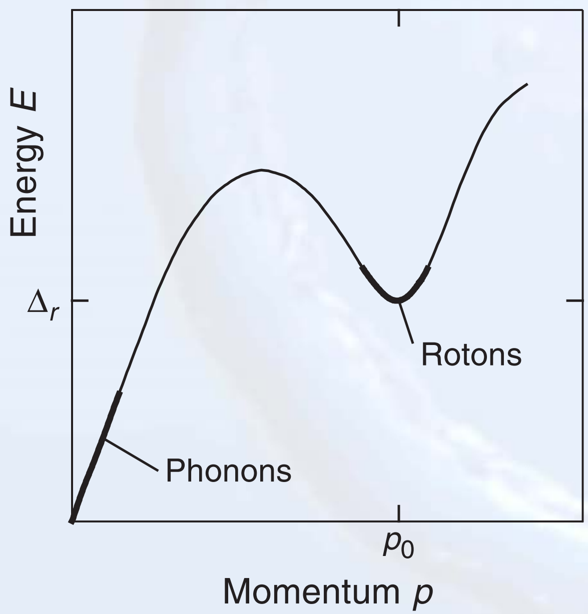
2.6 Excitation Spectrum of He-II: Landau Model

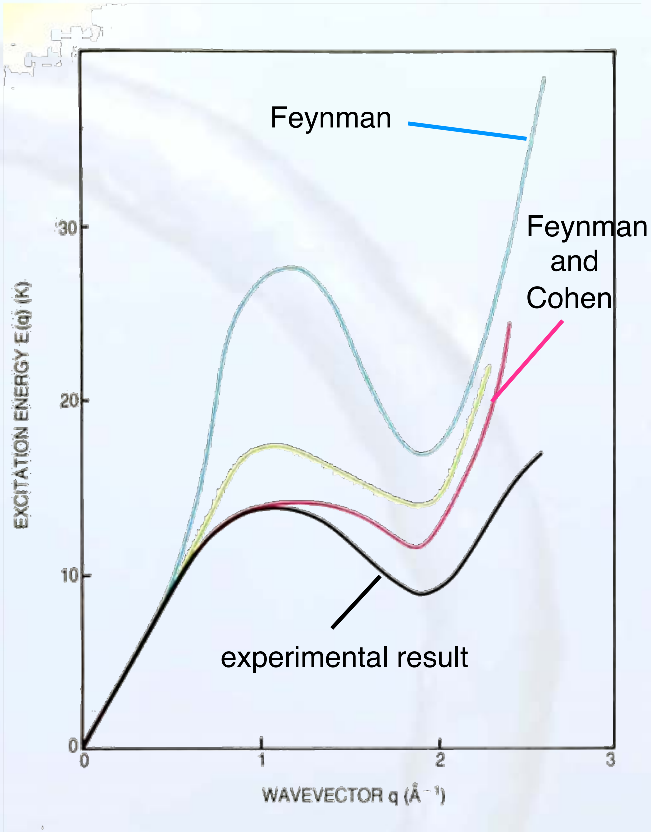
Landau's modification in 1947:

⟶ common dispersion curve

roton dispersion:

$$E = \Delta_r + \frac{(p - p_0)^2}{2m^*}$$





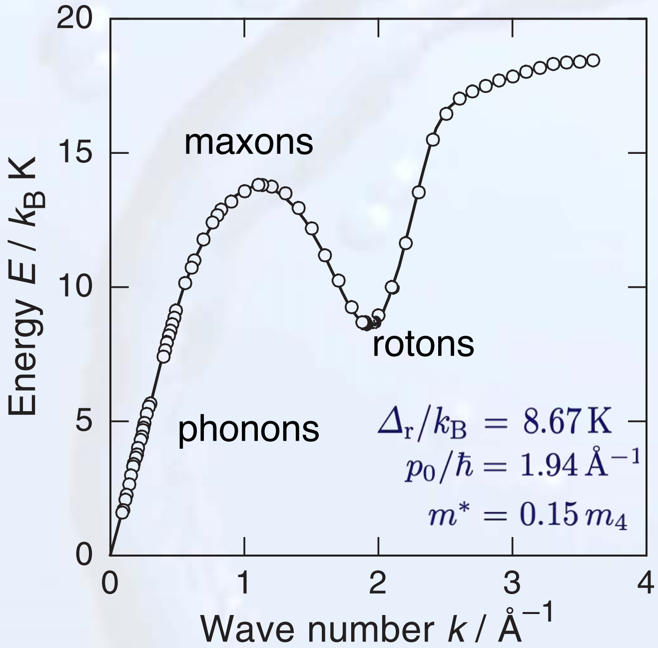
Feynman 1954:

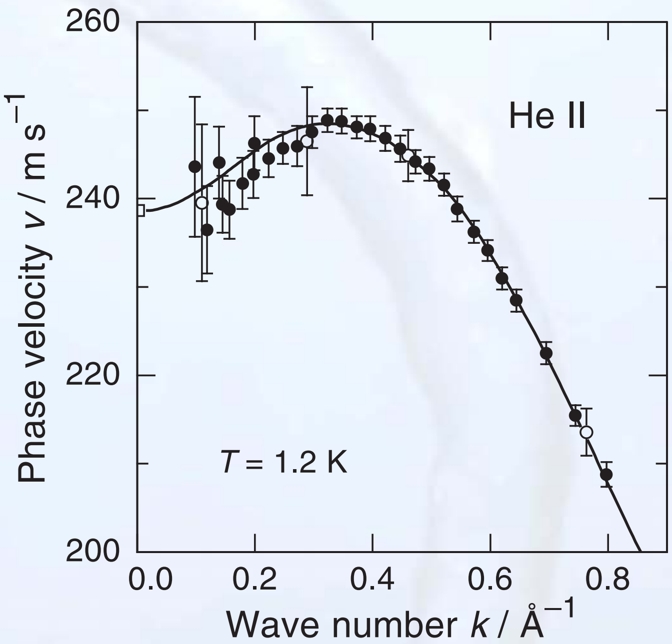
- QM calculation of dispersion curve from symmetry considerations
- improved by Feynman and Cohen in 1955

2.6 Excitation Spectrum of He-II: Landau Model

Experimental determination of the dispersion

Feynman's idea: inelastic neutron scattering





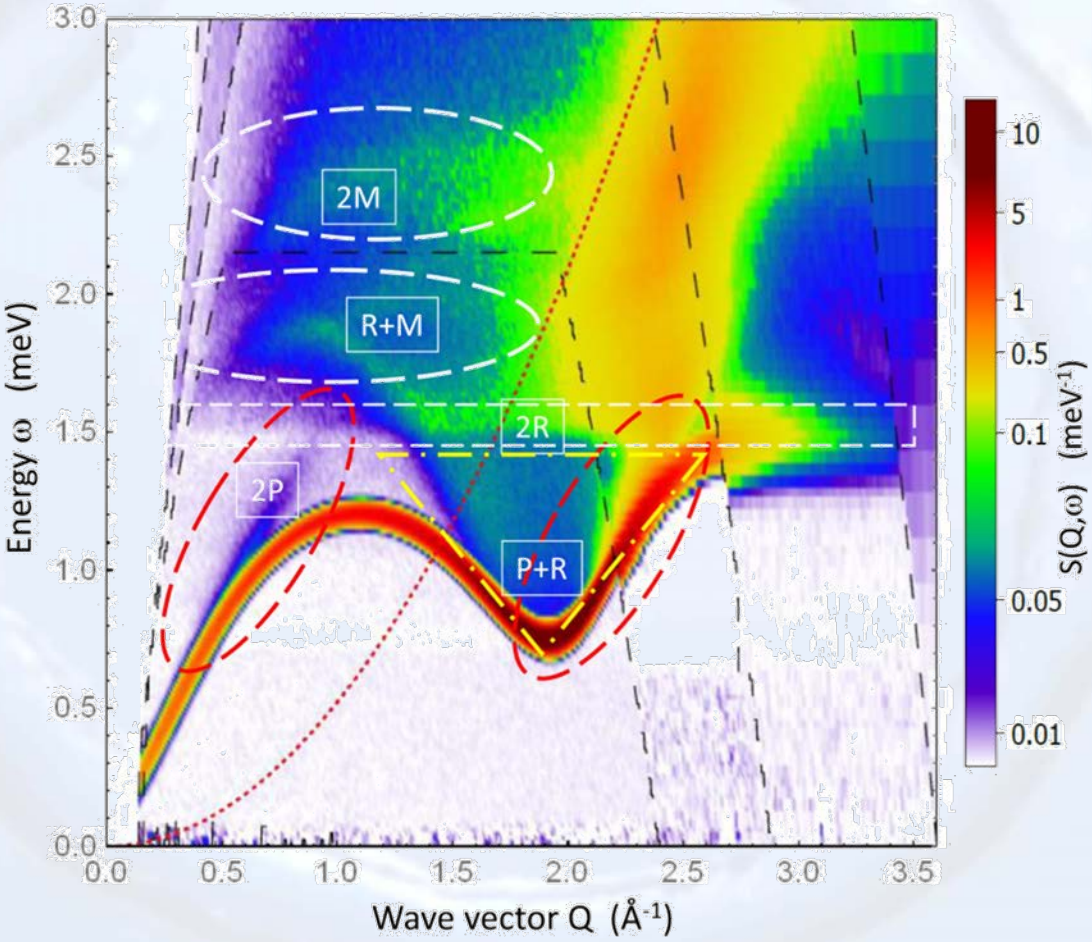
- good agreement with $q_{\min}$, $q_{\max}$
- linear dispersion with v = 238 m/s
- sharp excitations even at high q vectors
- single particle excitations are suppressed

- dispersion not perfectly linear
- anomaly at low wave vectors
  - ⟶ causes damping by three phonon scattering
  - ⟶ anomaly disappears at $p > 20$ bar

2.6 Excitation Spectrum of He-II: Landau Model

Experimental Determination of the Dispersion

new high-precision measurement



2.6 Excitation Spectrum of He-II: Landau Model

Normalfluid component:

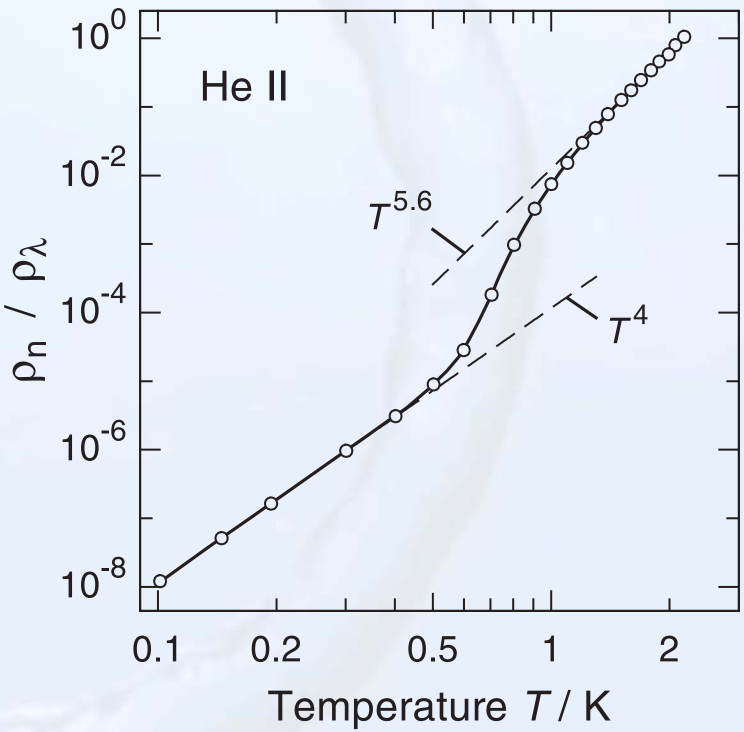
$$\varrho_{\mathrm{n,r}} = \frac{2\,p_0^4}{3\,\hbar^3}\sqrt{\frac{m^*}{(2\pi)^3 k_{\mathrm{B}} T}}\,\mathrm{e}^{-\Delta_{\mathrm{r}}/k_{\mathrm{B}}T}$$

Rotons

$$\varrho_{\mathrm{n}} = \varrho_{\mathrm{n,ph}} + \varrho_{\mathrm{n,r}}$$

$$\varrho_{\mathrm{n,ph}} = \frac{2\pi^2 k_{\mathrm{B}}^4}{45\,\hbar^3\, v_1^5}\,T^4$$

Phonons



- at low temperatures $\varrho_{\mathrm{n}} \propto T^4$ due to phonons
- rotons dominate between 0.5 K and 1.2 K
- above 1.2 K nature of excitations more complex

2.6 Excitation Spectrum of He-II: Landau Model

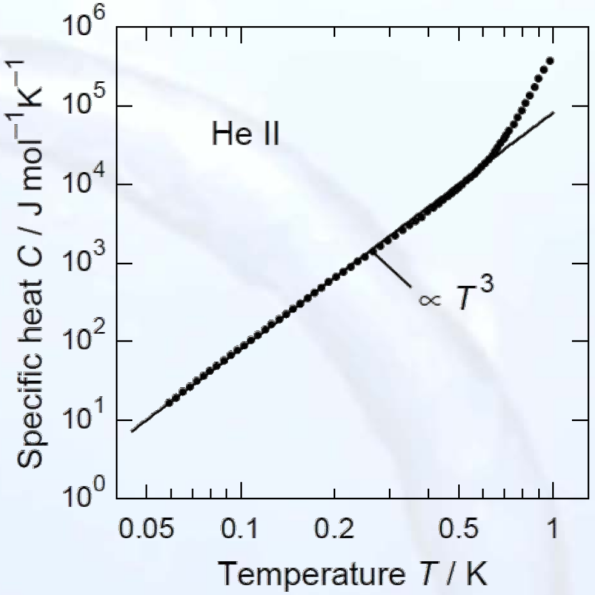
Specific heat:

a) low temperatures $T < 0.6\,\mathrm{K}$

only long wavelength phonons contribute

⟶ Debye model

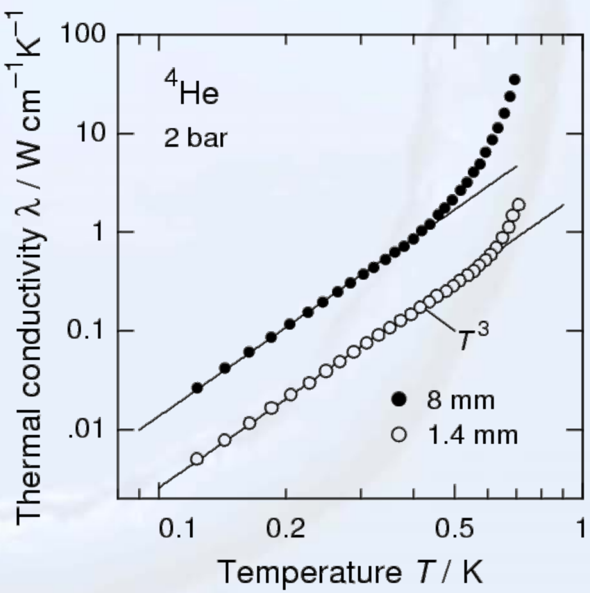
$$C_{\mathrm{ph}} = \frac{2\pi^2 k_{\mathrm{B}}^4}{15\varrho\hbar^3 v_1^3}\,T^3$$



measurement of thermal conductivity

Casimir regime $\ell = d$ — capillary cross section

$$\longrightarrow \quad \Lambda = \frac{1}{3} C_{\mathrm{ph}}\, v\, d \propto T^3$$



2.6 Excitation Spectrum of He-II: Landau Model

b) intermediate temperatures $\mathbf{0.6 < T < 1.2\,K}$

free energy $F_\mathrm{r} = -k_\mathrm{B} T n_\mathrm{r}$

$$n_\mathrm{r} = \frac{2p_0^2}{3\varrho\hbar^3}\sqrt{\frac{m^* k_\mathrm{B} T}{(2\pi)^3}}\,\mathrm{e}^{-\Delta_\mathrm{r}/k_\mathrm{B}T}$$

number density of rotons

$S_\mathrm{r} = -\partial F_\mathrm{r}/\partial T$

↓

$C_\mathrm{r} = T\partial S_\mathrm{r}/\partial T$

$$C_\mathrm{r} = \frac{2k_\mathrm{B}p_0^2}{3\varrho\hbar^3}\sqrt{\frac{m^* k_\mathrm{B} T}{(2\pi)^3}}\left\{\frac{3}{4} + \frac{\Delta_\mathrm{r}}{k_\mathrm{B}T} + \left(\frac{\Delta_\mathrm{r}}{k_\mathrm{B}T}\right)^2\right\}\mathrm{e}^{-\Delta_\mathrm{r}/k_\mathrm{B}T}$$

c) high temperatures $\mathbf{1.2\,K < T < T_\lambda}$

additional excitations contribute: maxons
lifetime of rotons becomes very short

⟶ excitations are not well-defined