2.5 Macroscopic Quantum State

→ quantization of circulation

→ Josephson effects

wave function of superfluid component

$$\psi(\boldsymbol{r}) = \psi_0\, \mathrm{e}^{\mathrm{i}\varphi(\boldsymbol{r})} \qquad (*) \qquad \text{with} \quad \psi^\star\psi = |\psi_0|^2 = \frac{\varrho_\mathrm{s}}{m_4}$$

m_4: mass of a ^{4}He atom

Schrödinger equation

$$-\mathrm{i}\hbar\nabla\psi = \boldsymbol{p}\,\psi$$

with $(*)$ ⟶ $\boldsymbol{p} = \hbar\nabla\varphi(\boldsymbol{r}) = m_4\boldsymbol{v}_\mathrm{s}$ ⟶

$$\boxed{\boldsymbol{v}_\mathrm{s} = \frac{\hbar}{m_4}\nabla\varphi(\boldsymbol{r})}$$

comment:

only valid at sufficiently low velocity were ϱ_s is constant

➡ $\boldsymbol{v}_\mathrm{s}$ determines the phase shift of wave function

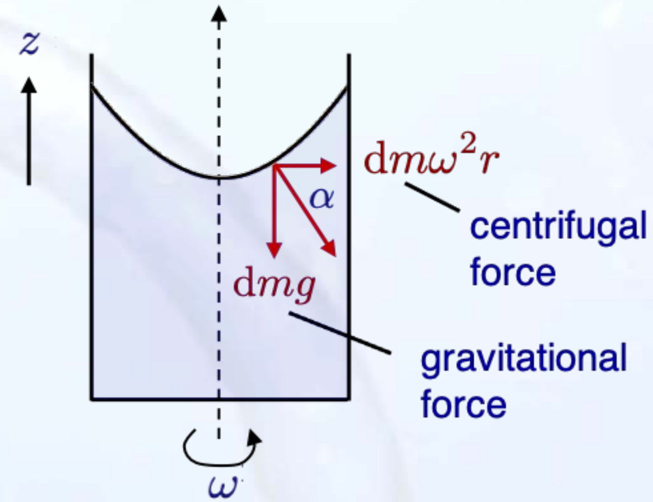
- $\boldsymbol{v}_\mathrm{s} = 0$ ⟶ phase is constant
- $\boldsymbol{v}_\mathrm{s} =$ const. ⟶ phase is changing uniformly

Interpretation

- phase is well-defined in entire liquid
- macroscopic wave function
- “rigid” coupling in momentum space

Proof of the concept: He-II under rotations

measurement of liquid meniscus



classical fluid ≙ normalfluid component ϱ_n

➡ solid body rotation $v_\mathrm{n} = \omega r$ — distance from axis of rotation

➡ profile of liquid surface ⟶ parabola

$$\tan\alpha = \frac{\mathrm{d}z}{\mathrm{d}r} = \frac{\omega^2 r}{g} \longrightarrow z = \frac{\omega^2}{2g} r^2$$

what about the superfluid component ?

two-fluid model $\mathrm{curl}\,\boldsymbol{v}_\mathrm{s} = 0$ **!**

for a simply-connected region this means every loop can be contracted to a point

$$\int_A \underbrace{\mathrm{curl}\,\boldsymbol{v}_\mathrm{s}}_{=0} \cdot \mathrm{d}\boldsymbol{f} = \oint_L \boldsymbol{v}_\mathrm{s} \cdot \mathrm{d}\boldsymbol{l} = 0$$

area enclosed by contour L

Stokes

- ▶ ϱ_s should not rotate (should be at rest)
- ▶ if so, centrifugal force is reduced

➡ $$z = \frac{\varrho_\mathrm{n}}{\varrho}\frac{\omega^2}{2g} r$$

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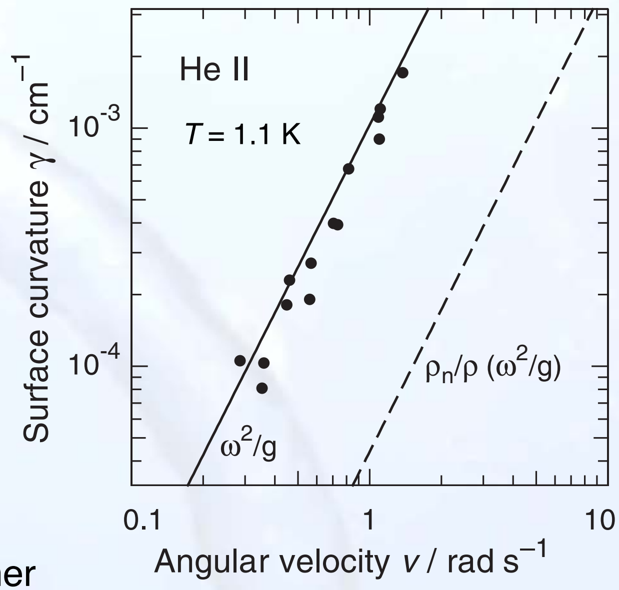
Experimental results



surface curvature: $\gamma = \omega^2/g$ all liquid

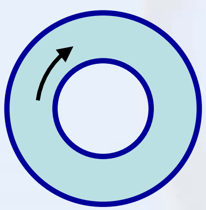
$\gamma = (\varrho_\mathrm{n}/\varrho)\omega^2/g$ only normalfluid

➡ curvature for all liquid is observed in Osborn experiment

Why is this the case?

⟶ let's do a thought experiment with an annular-shaped container

multiply-connected region



circulation:

$$\kappa = \oint_L \boldsymbol{v}_\mathrm{s} \cdot \mathrm{d}\boldsymbol{l} \qquad \boldsymbol{v}_\mathrm{s} = \frac{\hbar}{m_4}\nabla\varphi(\boldsymbol{r})$$

➡ $$\kappa = \frac{\hbar}{m_4}\Delta\varphi_L$$

- since $\psi(\boldsymbol{r})$ is a uniquely-defined function
  ⟶ phase can only be changed by $2\pi\, n$ for full cycle
- $\Delta\varphi = 2\pi\, n \qquad n = 0,\ 1,\ 2,\ 3,\ \ldots\ .$

circulation is quantized !

$$\kappa = \frac{h}{m_4}\, n$$

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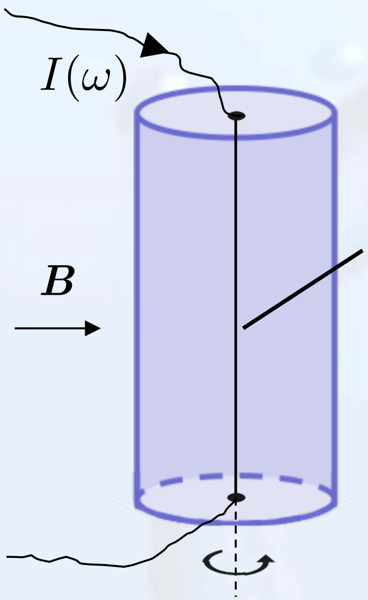
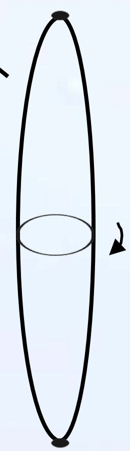
Experimental discovery of quantization of circulation

vibrating wire excited by current pules (Joe Vinen 1961)



thin wire
ø 2.5 μm,
5 cm long

transversal modes ≙ two circular polarized modes



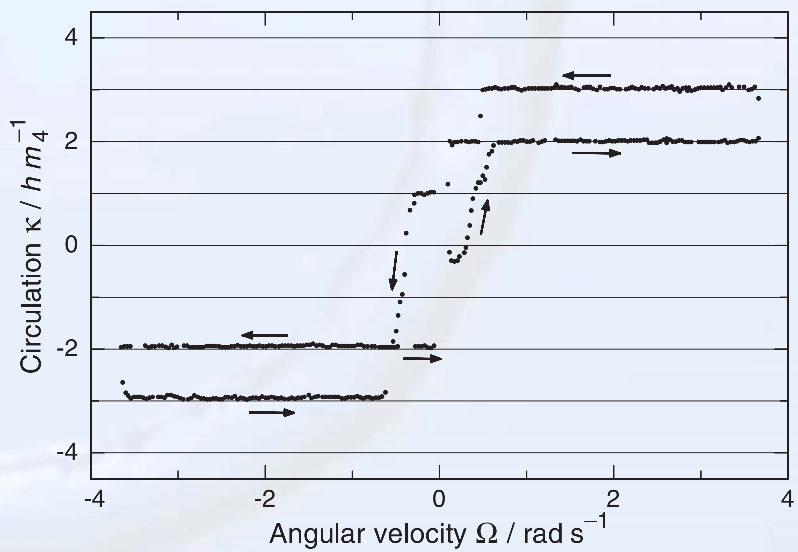
- without rotation: degenerate
- with rotation: lifting of degeneracy by Magnus force

frequency splitting: $\Delta\nu = \frac{\varrho_s}{2\pi\,\mathcal{M}}\,\kappa$

effective mass / length
(wire + ½ of expelled liquid)

experimental results

- quantization with expected value
- hysteresis effects are observed
- modern measurements up to $n = 4$



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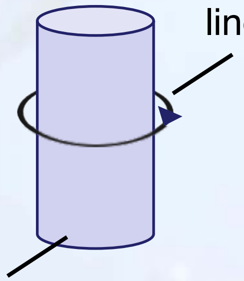
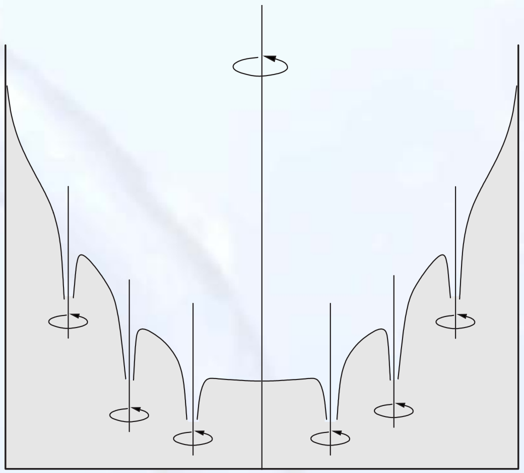
What has this to do with the rotation of bulk helium in a simply connected region?

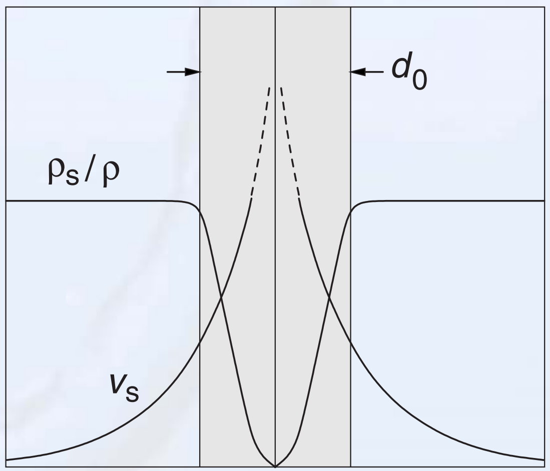
- vortices may occur with normal fluid core
- resulting in a multiply connected region





with $\kappa = \dfrac{h}{m_4} n$ and classical hydrodynamics one finds

$$v_s(r) = \frac{\kappa}{2\pi\, r} = \frac{1.58 \times 10^{-8}}{r} n \qquad \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$



- normal core: $v_s \rightarrow v_c$

$d_0 \approx 2 - 3\,\text{Å}$ $\triangleq$ coherence length
healing length

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Energy of a vortex

$$E_\mathrm{v} = \int_{a_0}^{b} \frac{\varrho_\mathrm{s} v_\mathrm{s}^2}{2} 2\pi r \, \mathrm{d}r$$

kinetic energy / volume

energy / length

a_0 : radius of vortex core

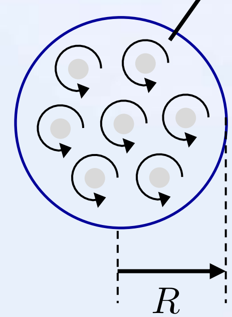
b : radius of vessel or ½ distance to next vortex

$$\kappa = v_\mathrm{s} \, 2\pi r \longrightarrow v_\mathrm{s}^2 = \frac{\kappa^2}{4\pi^2 r^2}$$

$$E_\mathrm{v} = \frac{\varrho_\mathrm{s} \kappa^2}{4\pi} \ln\left(\frac{b}{a_0}\right) \propto \kappa^2 \propto n^2$$

➡ vortex formation with $n = 1$ is preferred

N vortices



$$v_\mathrm{s}(R) = N \frac{h}{m^4} \frac{1}{2\pi R}$$

$N \propto R^2$ if evenly distributed

Why is not a large vortex forming?

➡ splitting up in many small vortices prohibits large kinetic energy in core of vortex near the axis of rotation (velocity at the edge of vessel is given)