4.6 Macroscopic Quantum Effects

macroscopic wave function

$$\left.\begin{array}{l}\Psi_{\alpha\beta}(\boldsymbol{r}) = \mathcal{A}_{\alpha\beta}(\boldsymbol{r})\,\mathrm{e}^{\mathrm{i}\varphi(\boldsymbol{r})} \\ \quad\quad\quad\quad \text{3 x 3 matrix}\end{array}\right\}$$

18 degrees of freedom

i) quantization of circulation

→ ^{4}He circulation is quantized

→ ^{3}He behavior is more complicated

^{3}He-A: ⟶ circulation is not quantized under ideal conditions, which means without external influences

⟶ $\boldsymbol{v}_\mathrm{s} \neq \dfrac{\hbar}{2m_3}\nabla\varphi$

- phase can be adjusted by modification of $\boldsymbol{l}$ ⟶ $\mathrm{curl}\,\boldsymbol{v}_\mathrm{s} = \dfrac{\hbar}{2m_3 r}\,\widehat{\boldsymbol{l}}\cdot\left(\dfrac{\partial\widehat{\boldsymbol{l}}}{\partial\phi}\times\dfrac{\partial\widehat{\boldsymbol{l}}}{\partial r}\right)$
- if $\boldsymbol{l}(\boldsymbol{r})$ ⟶ (continuous) vortices can be generated

  without singularity in $\boldsymbol{v}_\mathrm{s}$ and $\boldsymbol{l}(\boldsymbol{r})$

  for example near walls

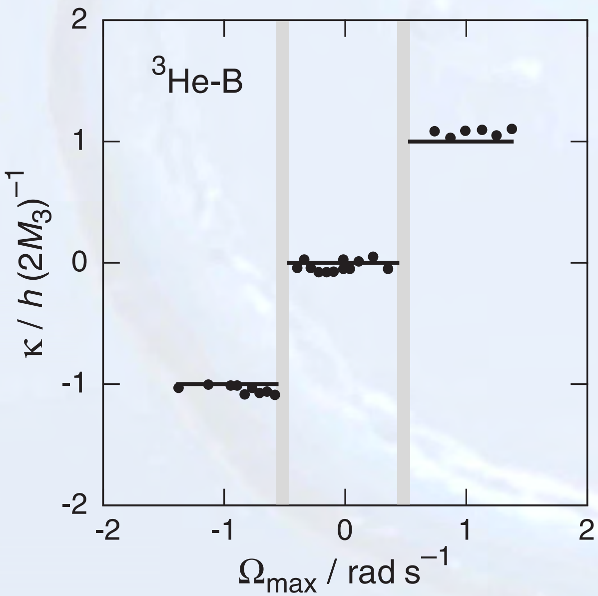
4.6 Macroscopic Quantum Effects

^{3}He-B: $\longrightarrow$ circulation is quantized

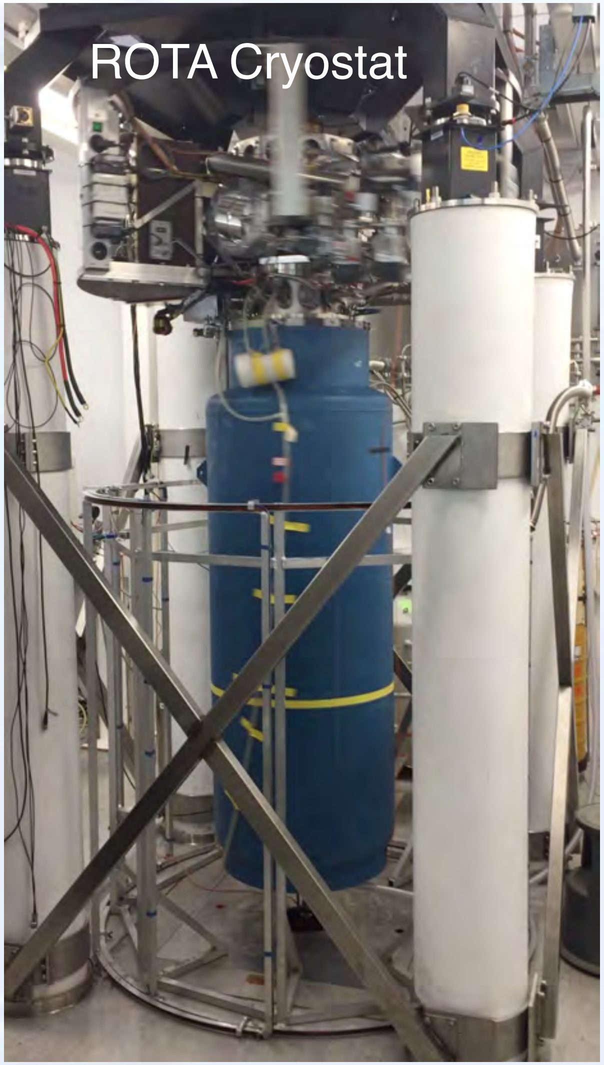
$$\longrightarrow \quad \boldsymbol{v}_\mathrm{s} = \frac{\hbar}{2m_3}\nabla\varphi$$

$$\longrightarrow \quad \boldsymbol{\kappa}_3 = \frac{h}{2m_3}$$

- Vinen-type experiment
- 1 rad/s = 0.16 revolutions /s

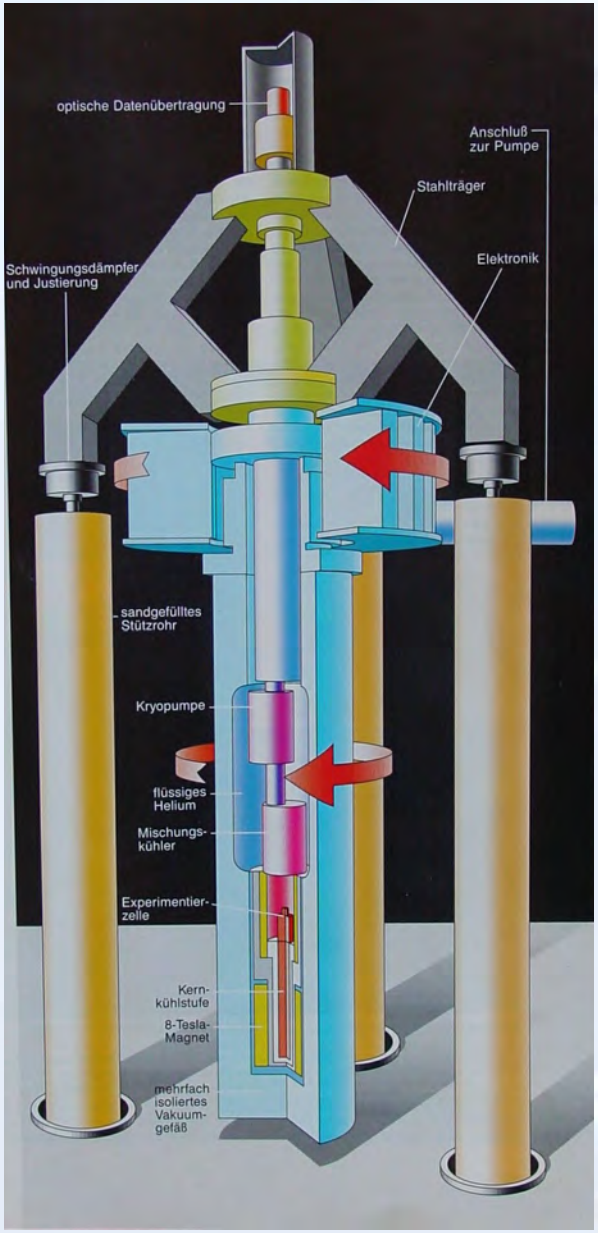


experimental problem:
rotation at very low temperatures



up to 3 revolutions / s

4.6 Macroscopic Quantum Effects



4.6 Macroscopic Quantum Effects

Quantized Vortices (structure much more complicated as in He-II)

^{3}He-A:

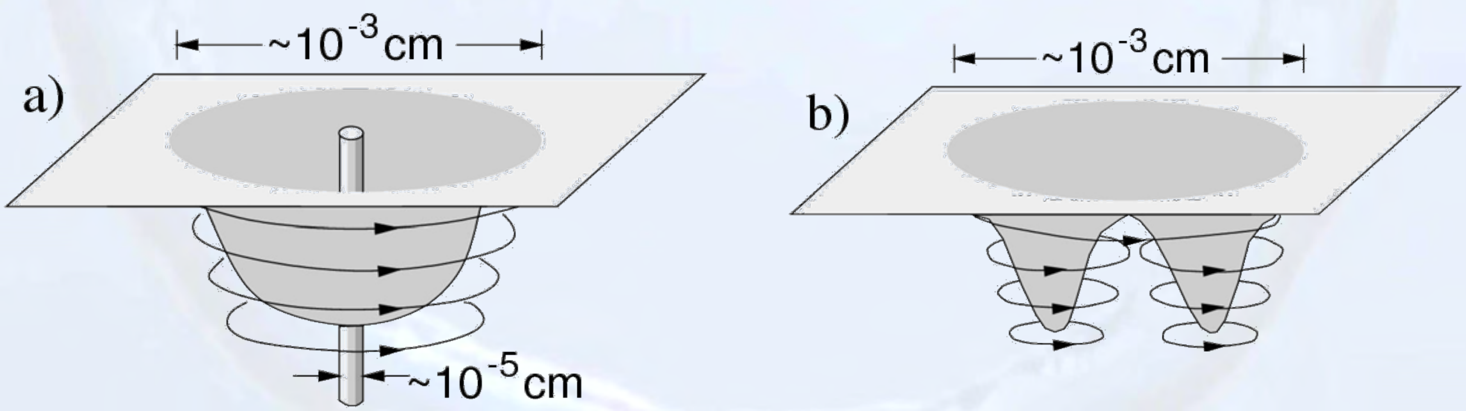
a) with uniform texture and orbital field $\boldsymbol{l}$ ⟶ vortices with normal-fluid hard core $\xi_0 \approx 100\,\mathrm{nm}$ (coherence length)

extended soft region $\xi_\mathrm{d} \approx 6\,\mu\mathrm{m}$ $n = 1$

dipole healing length it describes over which distance $\boldsymbol{d} \parallel \boldsymbol{l}$ recovers

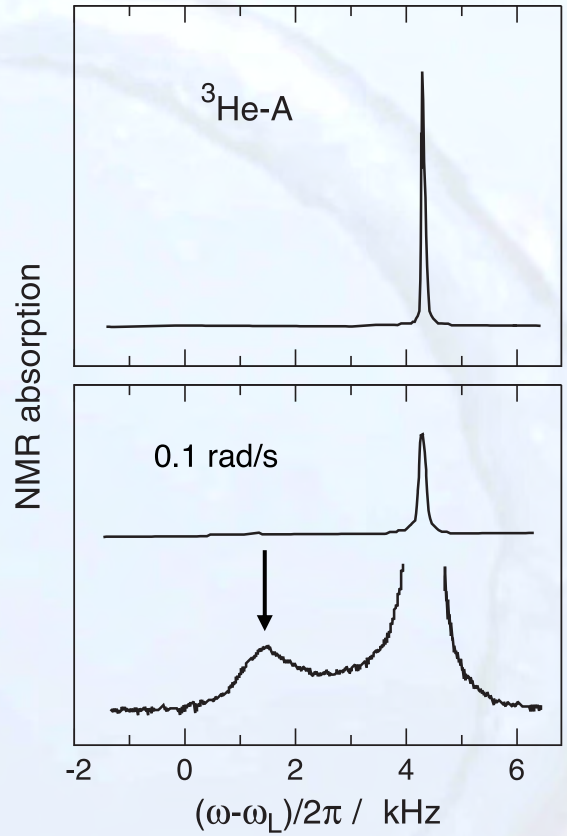
b) if $\boldsymbol{l}$ can adjust freely one finds continuous vortices with $n = 2$ without singularity (no hard core)

continuous velocity field



4.6 Macroscopic Quantum Effects

Investigation of vortices in ^{3}He-A with NMR



cryostat at rest

cryostat rotating

frequency shift because of localized spin waves in core!

container diameter 2.5 mm

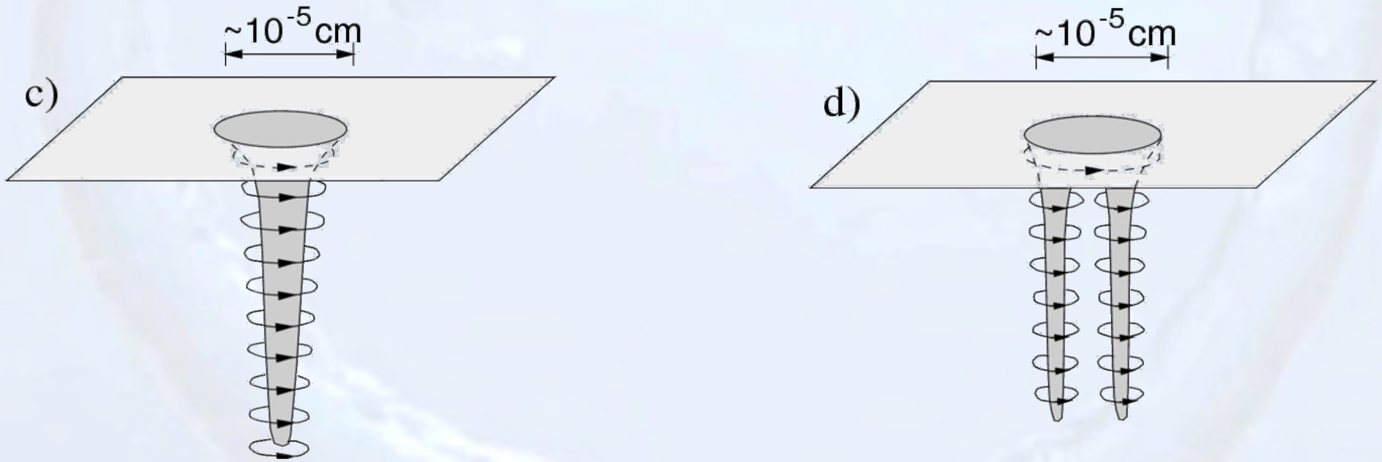
4.6 Macroscopic Quantum Effects

^{3}He-B: only vortices with hard core $\xi_0 \approx 10 \ldots\ 100\,\mathrm{nm}$

depends on pressure

c) single vortices with A phase in core

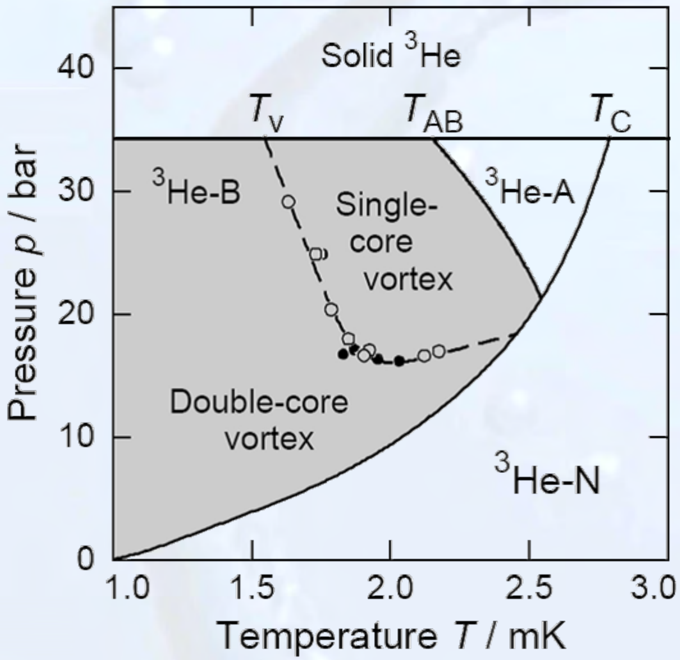
d) double vortices with two half-quantum of circulation and normal-fluid core



these vortices exist in distinct parts of the phase diagram

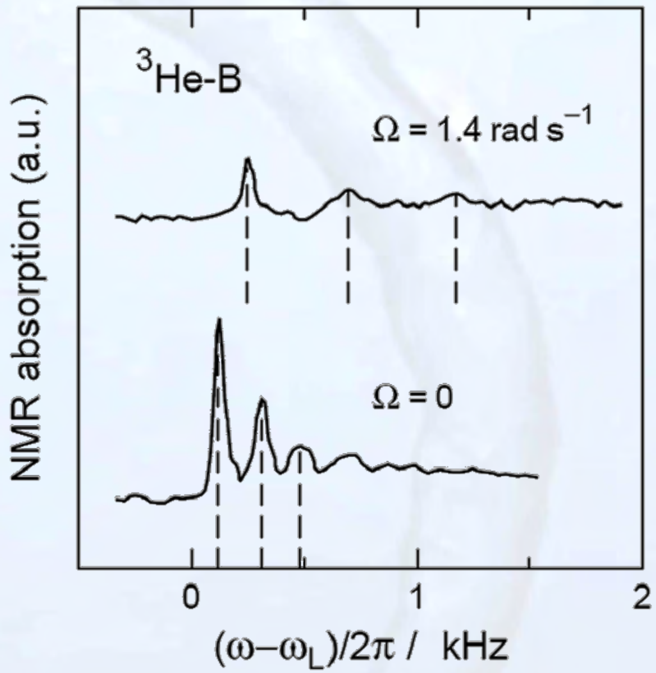
4.6 Macroscopic Quantum Effects

^{3}He-B: phase diagram under rotation



first order phase transition

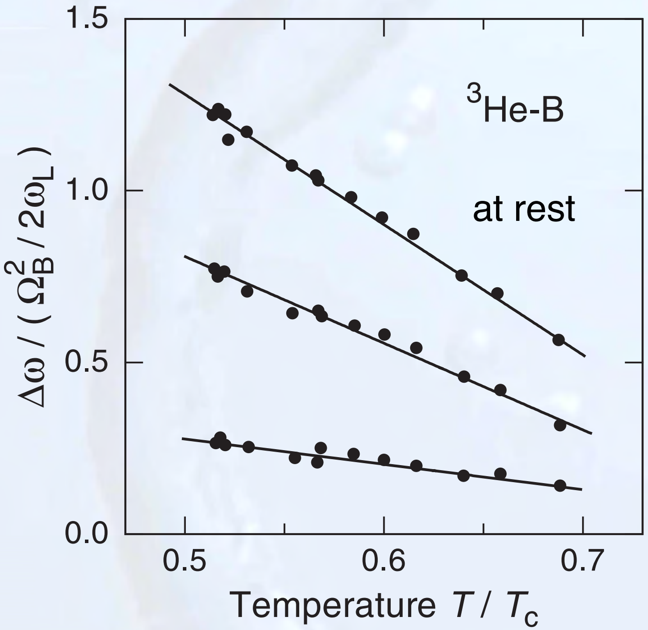
spin waves resonances (collision-less)



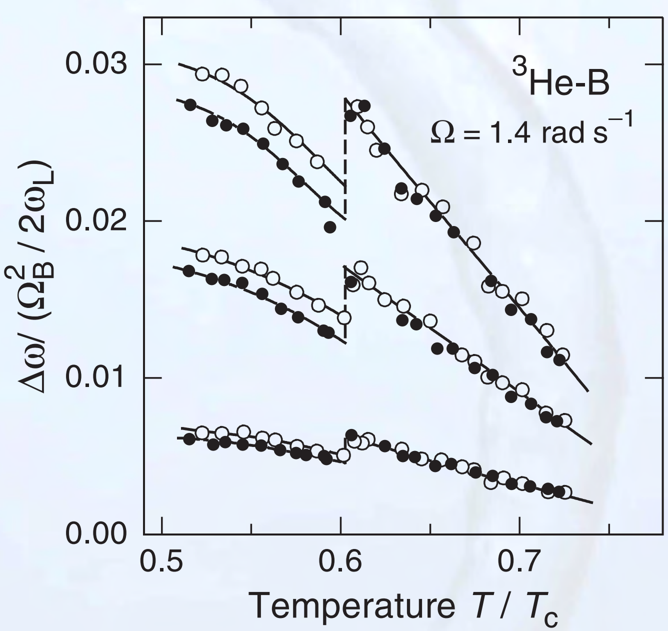
under rotation $\longrightarrow$ larger spacing
because additional term in free energy

4.6 Macroscopic Quantum Effects

^{3}He-B: phase diagram under rotation



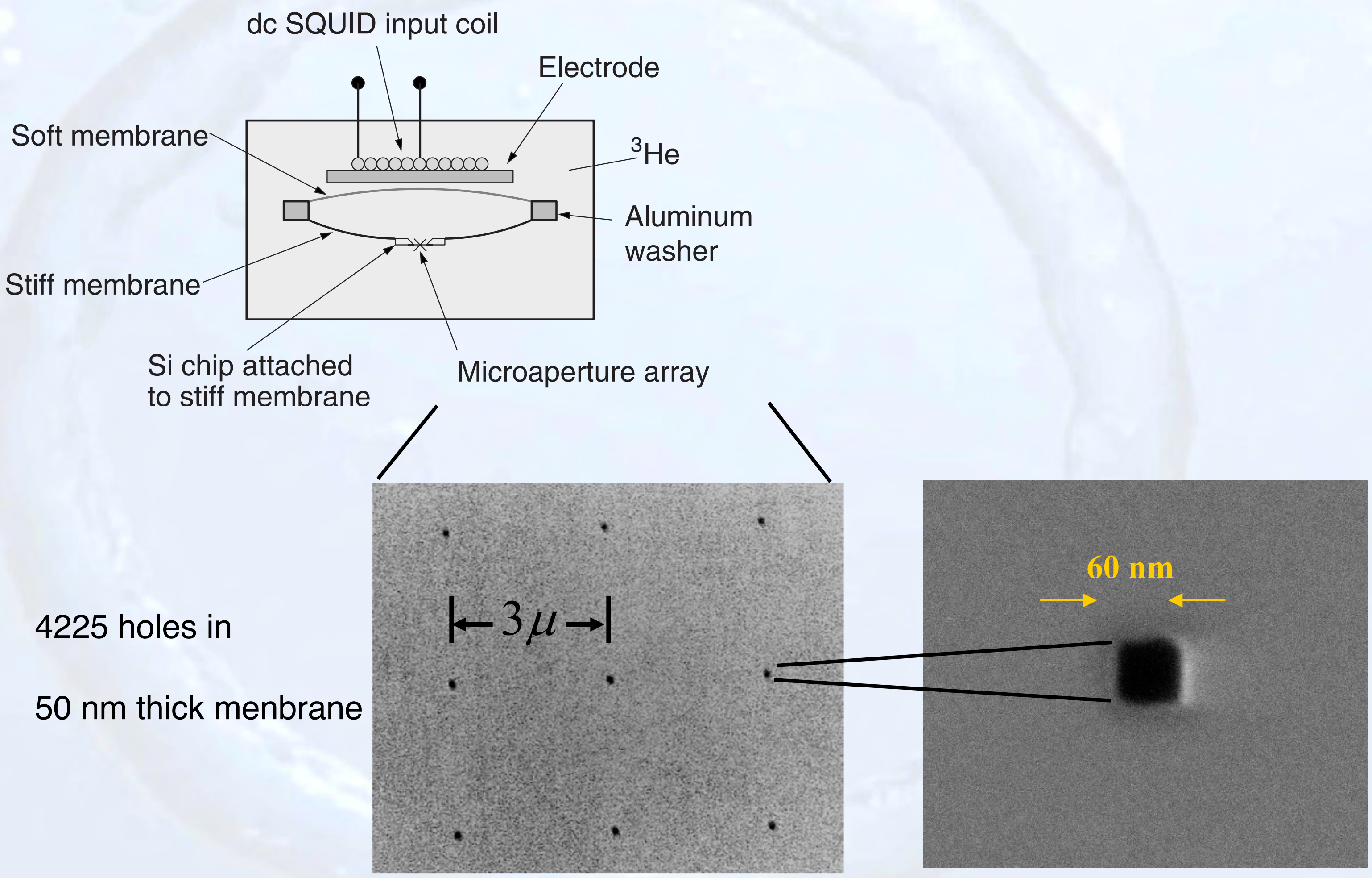
spin waves resonances (collision-less)



hysteresis is observed

⟶ 1st order transition

ii) Josephson effects



4225 holes in

50 nm thick menbrane

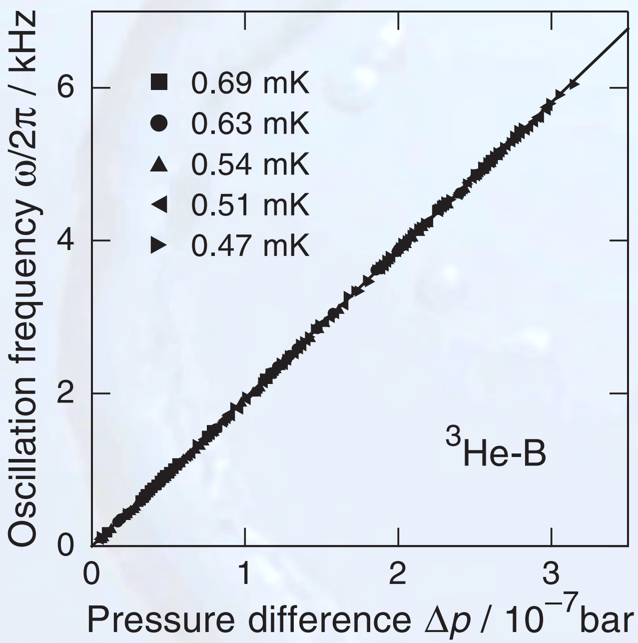
4.6 Macroscopic Quantum Effects

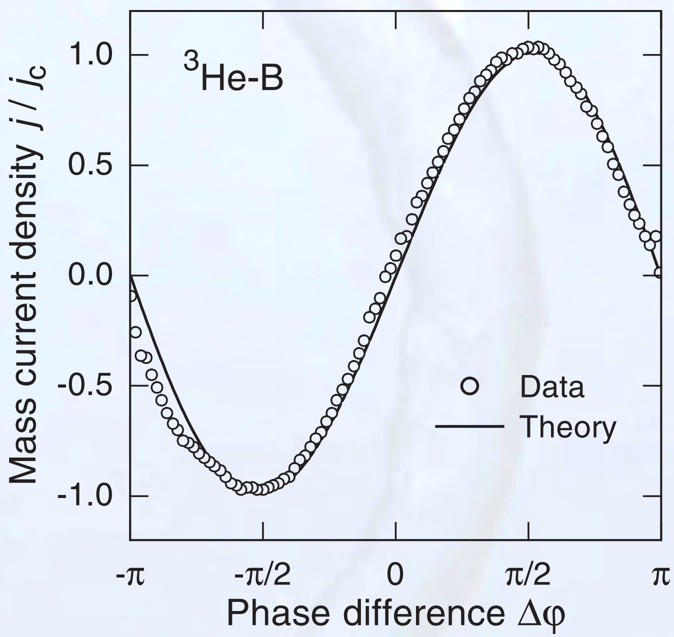
Josephson frequency

$$\omega_\mathrm{J} = m_3\,\Delta p/(\varrho\hbar)$$

Josephson dc current

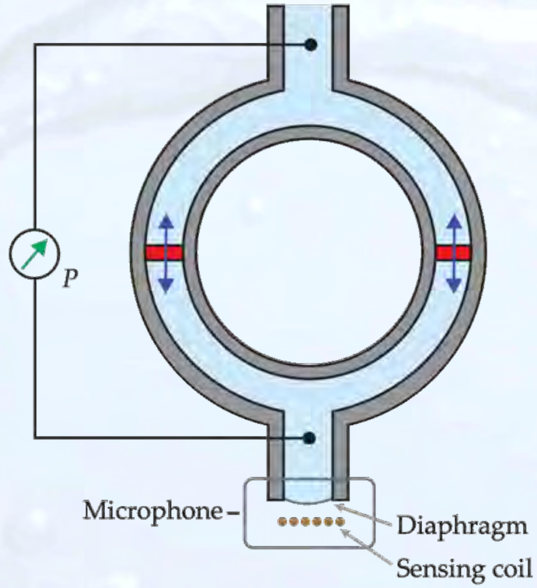
$$j = j_\mathrm{c}\sin(\Delta\varphi)$$

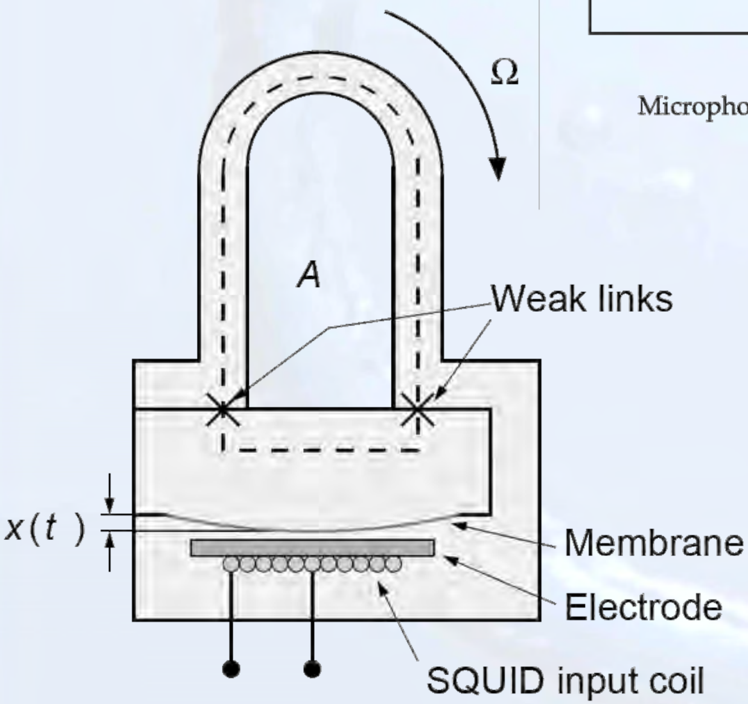




4.6 Macroscopic Quantum Effects

DC-SHeQUID: Superfluid He QUantum Interference Devices





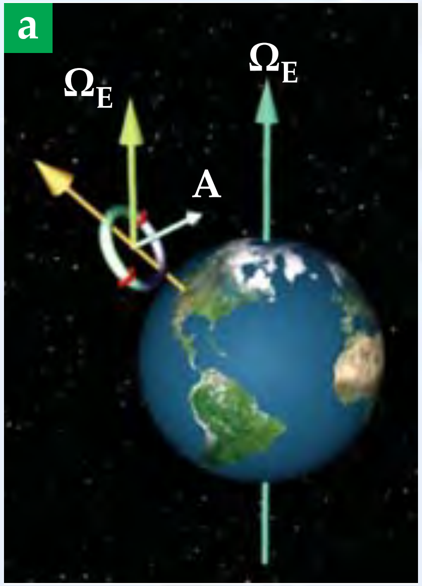
actual device

4.6 Macroscopic Quantum Effects

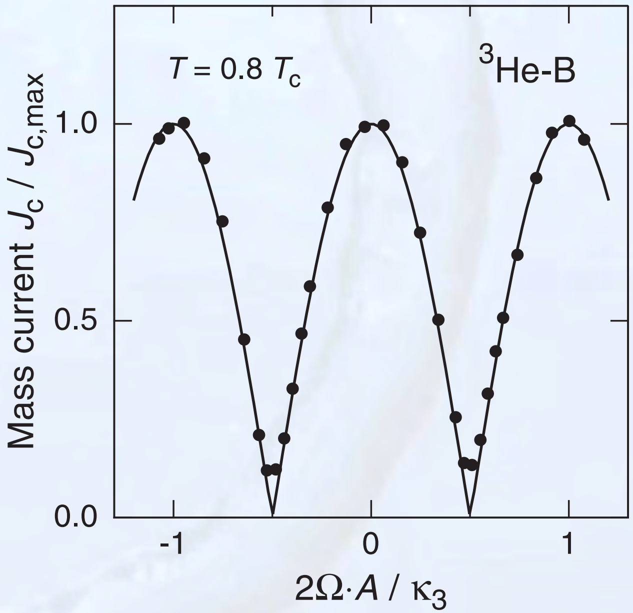
DC-SHeQUID in Earth rotation

angular velocity of rotating system (earth)

normal vector of loop



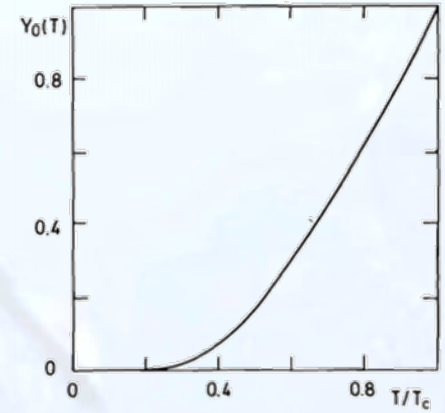
$$j_c = 2j_c^0 \left| \cos\left(\pi \frac{2\boldsymbol{\Omega}\cdot\boldsymbol{A}}{\kappa_3} \right) \right|$$



→ perfect agreement with theoretical expectations

4.7 Normalfluid Density

normalfluid density $\longleftrightarrow$ thermal excitations of quasi-particles



General expression from Landau's Fermi liquid theory

$$\overleftrightarrow{\varrho}_{\rm n} = \frac{m^*}{m}\left(\overleftrightarrow{1} + \frac{1}{3}F_1\frac{\overleftrightarrow{\varrho}_{\rm n,0}}{\varrho}\right)^{-1}\overleftrightarrow{\varrho}_{\rm n,0}$$

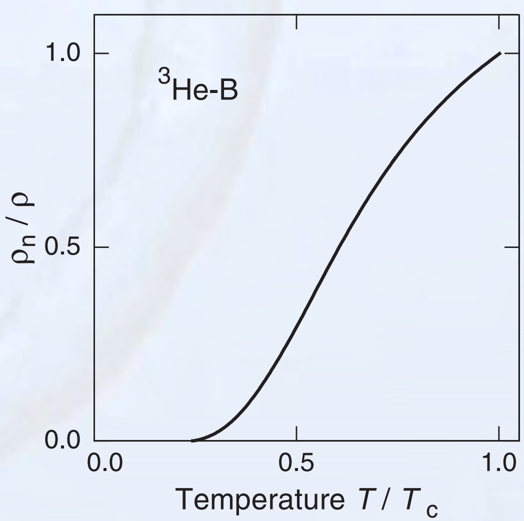
normalfluid density without Fermi liquid correction

$\longrightarrow$ temperature dependence given by Yosida function $Y_0(\widehat{\boldsymbol{k}}, T)$

^{3}He-B

isotropic $\longrightarrow$ independent of $\widehat{\boldsymbol{k}}$ $\longrightarrow$ $Y_0(\widehat{\boldsymbol{k}}, T) = Y_0$

Andronikasvili-like experiment



$$\varrho_{\rm n} = \varrho\frac{(1+F_1/3)\,Y_0}{1+F_1Y_0/3}$$

temperature dependent scalar!

- monotonic increase for $T \rightarrow T_{\rm c}$
- disappears exponentially for $T \rightarrow 0$

4.7 Normalfluid Density

^{3}He-A much more complicated situation!

close to T_c ϱ_n can approximated by:

$$\varrho_{\mathrm{n},\perp} - \varrho = 2\varrho_{\mathrm{n},\parallel} = -\frac{7}{5}\,\zeta(3)\,\frac{m}{m^*}\,\varrho\left(\frac{\Delta_\mathrm{m}}{\pi k_\mathrm{B} T_\mathrm{c}}\right)^2$$

parallel to orbital momentum

perpendicular to orbital momentum

very low temperatures:

$$\varrho_{\mathrm{n},\parallel} = \pi^2\,\frac{m}{m^*}\,\varrho\left(\frac{k_\mathrm{B}T}{\Delta_\mathrm{m}}\right)^2 \qquad \propto T^2$$

$$\varrho_{\mathrm{n},\perp} = \frac{7}{15}\pi^4\,\frac{m}{m^*}\,\varrho\left(\frac{k_\mathrm{B}T}{\Delta_\mathrm{m}}\right)^4 \qquad \propto T^4$$

Specific heat

^{3}He-B $$C_\mathrm{B}(T) = \sqrt{2\pi}\,D(E_\mathrm{F})\,k_\mathrm{B}\Delta_\mathrm{B}\left(\frac{\Delta_\mathrm{B}}{k_\mathrm{B}T}\right)^{3/2} \mathrm{e}^{-\Delta_\mathrm{B}/(k_\mathrm{B}T)}$$

^{3}He-A $$C_\mathrm{A}(T) = \frac{7}{5}\pi^2\left(\frac{T}{\Delta_\mathrm{m}}\right)^2 C_\mathrm{N}(T) \propto T^3$$ for $T \ll \Delta_\mathrm{m}/k_\mathrm{B}$ and $\widehat{\boldsymbol{k}} = \pm\widehat{\boldsymbol{l}}$

$$C = \frac{m^*}{m}\,C_\mathrm{FG}$$

4.8 Collective Excitations — Sound Propagation

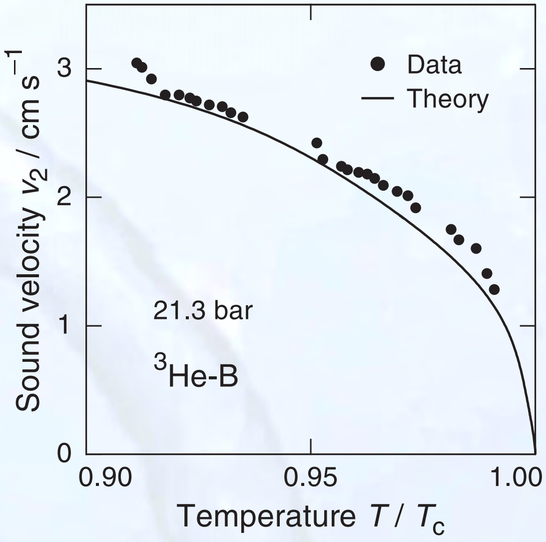
a) 2nd Sound

^{3}He-B

$$v_2 = \sqrt{\frac{\varrho_s}{\varrho_n} S^2 \left(\frac{\partial T}{\partial S}\right)_\varrho}$$

as in case of He-II

- v_2 just a few cm/s
- reduction $\propto \dfrac{T}{T_F}$ in S

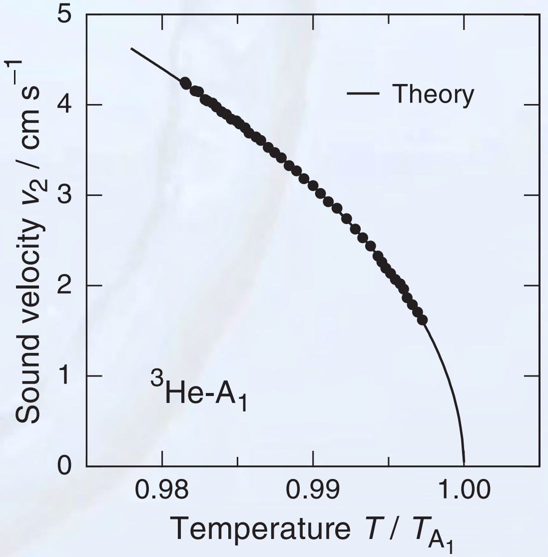


^{3}He-A$_1$

$$v_2 = \frac{\gamma\hbar}{2m^*}\sqrt{\frac{\varrho}{\chi}\,\frac{\varrho_{s,\perp}}{\varrho_{n,\perp}}}$$

not only entropy wave but also spin wave

→ higher velocity as in case of ^{3}He-B

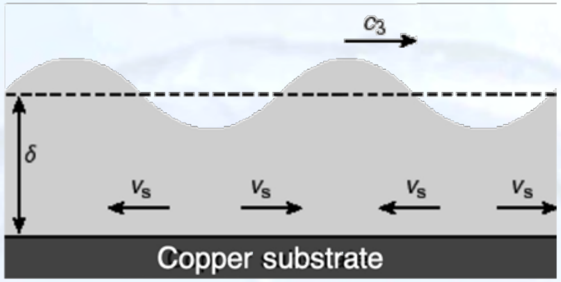


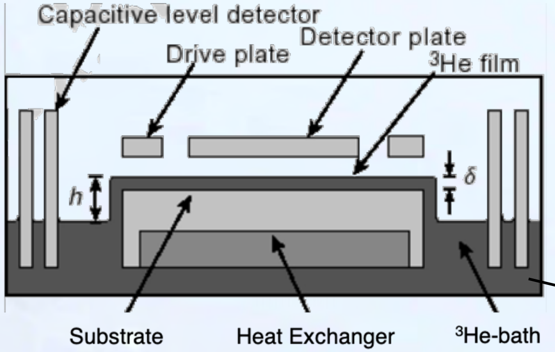
4.8 Collective Excitations — Sound Propagation

b) 3rd Sound

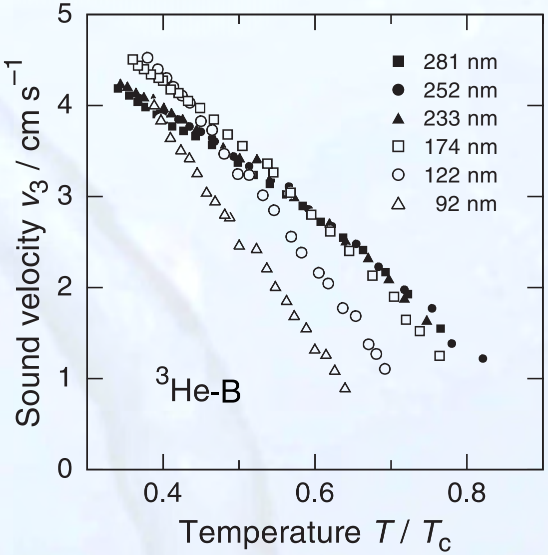
^{3}He-B

$$v_3 = \sqrt{\frac{\langle \varrho_s \rangle}{\varrho} \frac{3\alpha}{d^3}}$$





$\varepsilon = 1.0426$

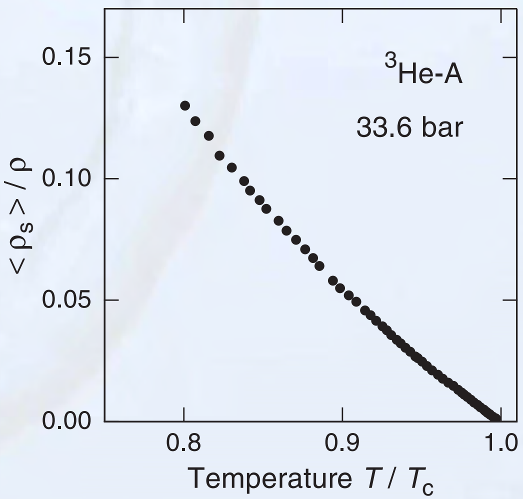


c) 4th Sound

$$v_4 = v_1 \sqrt{\frac{\langle \varrho_s \rangle}{\varrho} \frac{5}{3} [2 - \cos^2 \Theta)]}$$

$$\langle \varrho_s \rangle / \varrho \propto (1 - T/T_c)$$

angle between q and l



d) order parameter modes

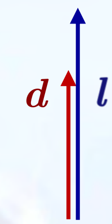
⟶ collective excitations of Cooper pairs $\hbar\omega < 2\Delta$

- relative motion of $\boldsymbol{d}$ and $\boldsymbol{l}$
- inner structure of Cooper pairs

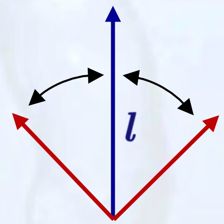
⟶ 18 different order parameter modes

$\hbar\omega > 2\Delta$ ⟶ pair breaking

example $\boldsymbol{d} \parallel \boldsymbol{l}$



$\boldsymbol{d}$ splits up in two perpendicular components

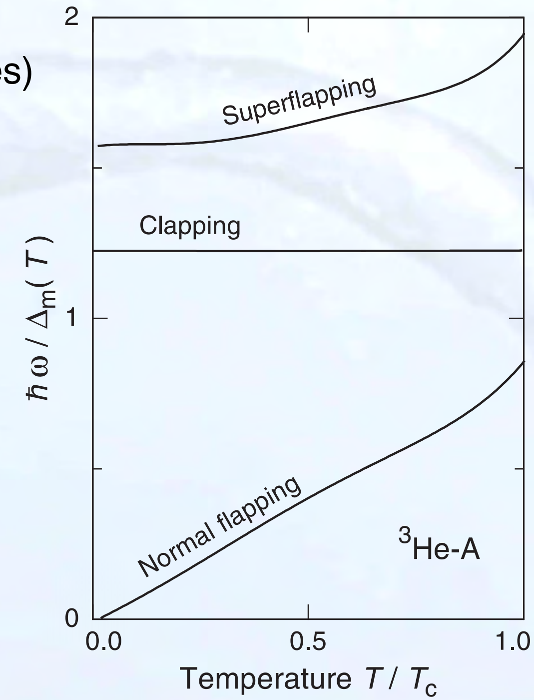


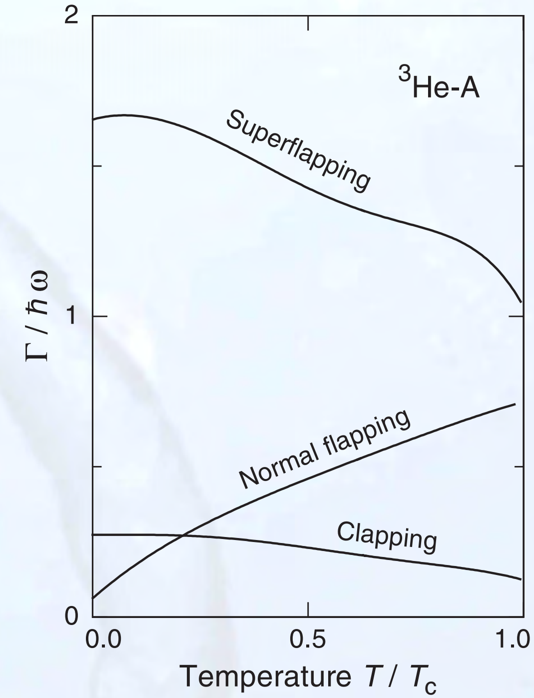
^{3}He-A (examples of order parameter modes)

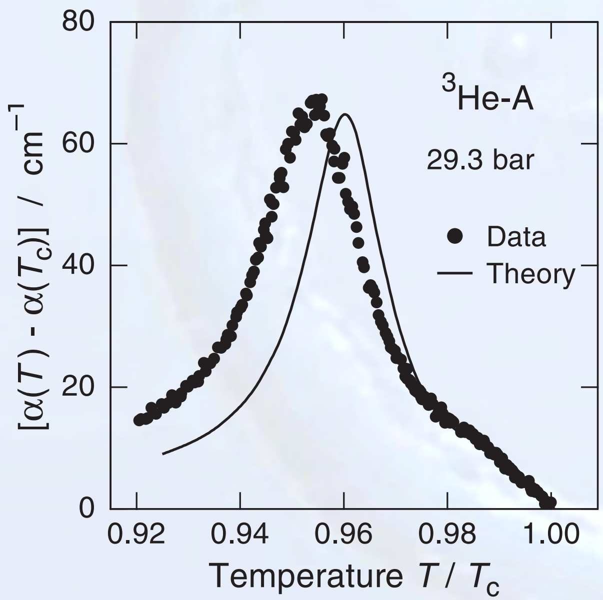
name	energy $(\hbar\omega/\Delta_{\mathrm{m}}(T))$		
normal flapping	$\propto \sqrt{\frac{4}{5}\frac{T}{T_{\mathrm{c}}}}$		
clapping	1.23		
superflapping	1.56	for	$T \to 0$
	2	for	$T \to T_{\mathrm{c}}$

4.8 Collective Excitations — Order parameter modes

^{3}He-A (examples of order parameter modes)





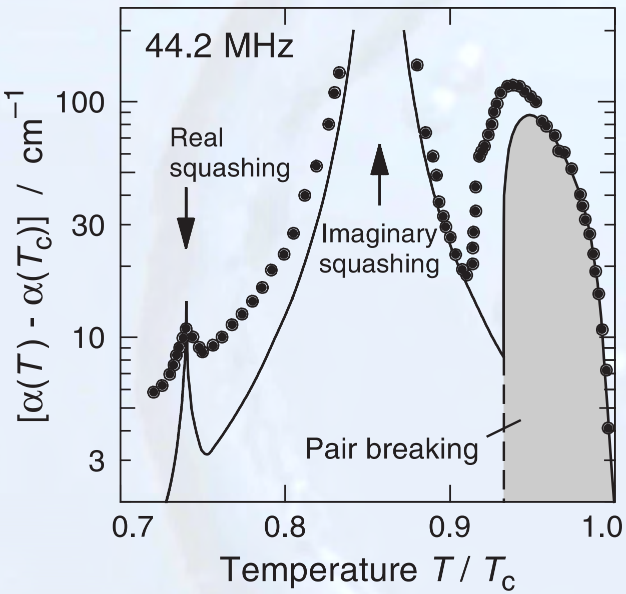


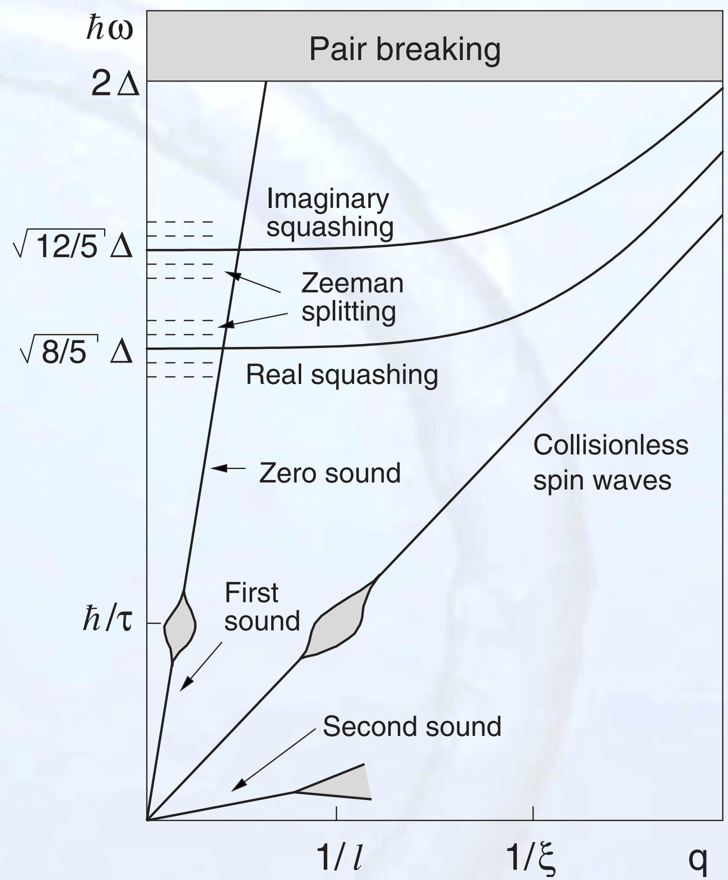
- damping of longitudinal zero sound ^{3}He-A
- clapping resonance

4.8 Collective Excitations — Order parameter modes

^{3}He-B (classification $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$, J_z)

dispersion relation





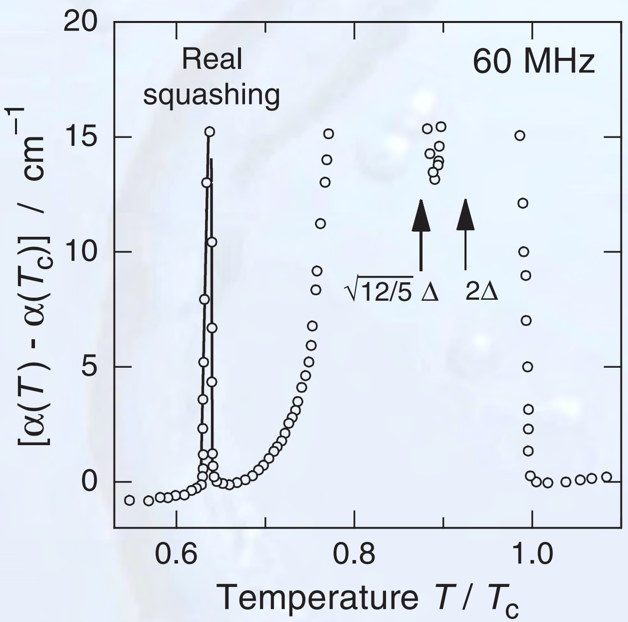
- since gap decreases with temperature
  ⟶ with fixed sound frequencies, several modes can be excited at different temperatures

transition into collision-less regime

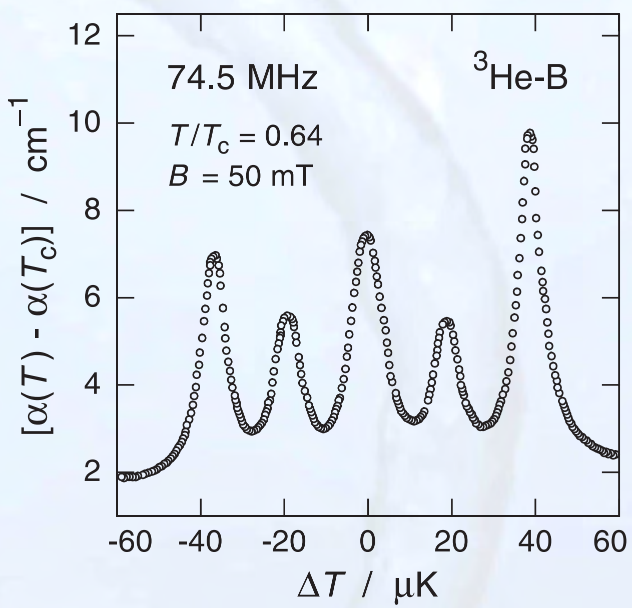
- arrows indicated expected peak position

4.8 Collective Excitations — Order parameter modes

measurement at higher frequency 60 MHz



real squashing mode in magnetic field



- pair breaking just below T_c
- extremely sharp resonances at low temperatures

- $J = 2 \longrightarrow$ multiplicity $2J + 1 \longrightarrow$ 5 levels
- Zeeman splitting in magnetic field