I. Flavor in the Standard Model

In this lecture the phenomenology of quarks and leptons starting from their fundamental interactions is discussed.

In the Standard Model of particle physics, transitions between different species of quarks and leptons ("flavors") are mediated by the weak interaction. The strong interactions plays an important role when quark transitions are discussed, since quarks are bound into hadrons.

In theoretical scenarios beyond the Standard Model, new flavor transitions can arise. Quark and lepton observables are therefore excellent testing-grounds for **New Physics**.

Sensitive observables in quark and lepton-flavor physics are:
Rates of rare, flavor-changing neutral current (FCNC) decays; → new results
Mixing (flavor or meson mixing); → recent results
CP violation and angular observables; → new results
Search for lepton-flavor violation; → new experiments
Anomalous magnetic moments; → new results
Electrical dipole moments; → new results

very active research field

Two very recent examples in public media:

https://www.faz.net/aktuell/wissen/physik-mehr/unerwartetes-messergebnis-neue-physik-am-teilchenhorizont-17265488.html

Large Hadron Collider : Neue Physik am Teilchenhorizont?
- Von Manfred Lindinger
- -Aktualisiert am 26.03.2021-18:47

https://www.spiegel.de/wissenschaft/natur/myon-g-2-experiment-neue-erkenntnisse-in-der-teilchenphysik-a-f1d11ab8-a63e-4571-83c6-c148704ae079

Kundschafter ins Unbekannte

Seit 50 Jahren ersehnen Forscher Einblicke in die Welt jenseits der bekannten Naturgesetze. Jetzt öffnet sich das Tor zu einer neuen Physik.

Von Johann Grolle
07.04.2021, 16.48 Uhr

1. Fundamental particles and gauge symmetries

There exists 3 generations of fundamental particles:

	I	II	III	Q_{el}
Quarks	u d	c s	t b	$+\frac{2}{3}$ $-\frac{1}{3}$
Leptons	ν_e e	ν_μ μ	ν_τ τ	0 -1

The Standard Model describes the strong, weak and electromagnetic force between the fundamental particles. The interactions satisfy the gauge symmetry

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

While the 3 generations carry the same charges under the gauge group, the particles within each generation are classified according to their interaction.

Quarks: $Q_L^i\left(3,\ 2,+\tfrac{1}{6}\right)$ $U_R^i\left(3,\ 1,+\tfrac{2}{3}\right)$ $D_R^i\left(3,\ 1,-\tfrac{2}{3}\right)$

Leptons: $L_L^i\left(1,\ 2,-\tfrac{1}{2}\right)$ $E_R^i\left(1,\ 1,-1\right)$

Notation: Q_L^i ← generation

↑ chirality

is a triplet under $SU(3)_C$, a doublet under $SU(2)_L$
and carries an U(1)-hypercharge Y=+1/6 ($Q = Y + T_3$)

The 3 generation, i=1…3 , of fermions differ only in the masses.

Quarks:

$$Q_L^i = \left\{ \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right\}$$

$$U_R^i = (u_R, c_R, t_R)$$

$$D_R^i = (d_R, s_R, b_R)$$

Leptons:

$$L_L^i = \left\{ \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix} \right\}$$

$$E_R^i = (e_R \mu_R, \tau_R)$$

2. Yukawa coupling to Higgs and the origin of mass

Standard Model
Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$\mathcal{L}_{Gauge}$ <u>gauge or kinetic term describes dynamics of fermions</u>

$$\mathcal{L}_{Gauge} = \sum_{f} i\bar{\psi}_f \gamma_\mu D^\mu \psi_f \qquad \psi_f = \{Q_L, U_R, D_R, L_L, E_R\} \qquad \bar{\psi}_f = \psi_f^\dagger \gamma^0$$

adjoint spinor

With the covariant derivative

$$D^\mu = \partial^\mu + \underbrace{ig_s G_a^\mu T_A} + \underbrace{igW_b^\mu \tau_b} + \underbrace{ig' B^\mu Y}$$

8 gluon fields and
SU(3) generators,
acting on fermion
color triplets

Gell-Mann
matrices

3 weak boson fields
and SU(2) generators,
acting on fermion
isospin doublets

Pauli
matrices

1 hypercharge boson
$U(1)_y$ charge operator

Example: Gauge terms for left-handed quarks and leptons

$$\mathcal{L}_{Gauge}^{Quarks} = i\bar{Q}_L^i \gamma_\mu \left(\partial^\mu + ig_s G_a^\mu \frac{\lambda_a}{2} + igW_b^\mu \frac{\sigma_b}{2} + ig'B^\mu \frac{1}{6} \right) Q_L^i$$

$$\mathcal{L}_{Gauge}^{Lepton} = i\,\bar{L}_L^i \gamma_\mu \left(\partial^\mu + \qquad igW_b^\mu \frac{\sigma_b}{2} - ig'B^\mu \frac{1}{2} \right) L_L^i$$

(expressions for right-handed quarks and leptons less interesting: only NC – why?)

$\mathcal{L}_{Higgs}$ Higgs-potential describing the scalar self-interactions of Higgs field

Higgs-field: $H\left(1,\ 2,+\tfrac{1}{2}\right)=\begin{pmatrix} h^{+} \\ \tfrac{1}{\sqrt{2}}\left(v+h(x)\right)\end{pmatrix}$ with the non-zero vacuum expectation value (VEV) v of the Higgs field

$$\mathcal{L}_{Higgs}=\mu^{2}H^{\dagger}H-\tfrac{\lambda}{4}\left(H^{\dagger}H\right)^{2}=-V\left(H\right)$$

VEV breaks the electroweak gauge symmetry: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

$\mathcal{L}_{Yukawa}$ Yukawa interaction with Higgs field (w/ coupling Y)

$$\mathcal{L}_{Yukawa} = -\sum_{i,j}\left\{Y_U^{ij}\bar{Q}_L^i U_R^j \tilde{H} + Y_D^{ij}\bar{Q}_L^i D_R^j H + Y_E^{ij}\bar{L}_L^i E_R^j H + h.c.\right\}$$

where $$\tilde{H}\left(1,\ 2,-\tfrac{1}{2}\right) = \begin{pmatrix} \tfrac{1}{\sqrt{2}}\left(v + h^*(x)\right) \\ -h^- \end{pmatrix}$$

Y_U, Y_D, Y_E are 3×3 matrices in flavor space and describe the coupling to the Higgs field. After symmetry breaking the interaction with the VEV of the Higgs fields generate the fermion masses ($\mathcal{L}_{Yukawa} \rightarrow \mathcal{L}_{Mass}$):

$$\mathcal{L}_{Mass} = -\tfrac{v}{\sqrt{2}}\left\{\bar{U}_L^i Y_U^{ij} U_R^j + \bar{D}_L^i Y_D^{ij} D_R^j + \bar{E}_L^i Y_E^{ij} E_R^j + h.c.\right\} \quad \text{with } Q_L^i = \begin{Bmatrix} U_L^i \\ D_L^i \end{Bmatrix}$$

The Yukawa matrices describing the Yukawa interaction are in general complex and non-diagonal → flavor structure of the Standard Model

Which effects are already imprinted into the Yukawa matrices?

While $\mathcal{L}_{Gauge}$ exhibits a global flavor symmetry,

$$G_F = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$$

i.e, the gauge interaction are unchanged by linear transformations $Q_L \rightarrow V_Q Q_L$ with $V^+_Q V_Q = 1$, this symmetry is broken by the, in general non-diagonal, Yukawa couplings.

It is convenient to choose a flavor basis for the fermion fields in which the mass terms from $\mathcal{L}_{Mass}$ are diagonal. This can be achieved by unitary transformations:

$$U_R \rightarrow V_{u_R} u_R \qquad U_L \rightarrow V_{u_L} u_L \qquad D_R \rightarrow V_{d_R} d_R \qquad D_L \rightarrow V_{d_L} d_L$$ (indices dropped)

With $V^\dagger_{A,q} V_{A,q} = 1$ and $V^\dagger_{u_L} Y_U V_{u_R} = \hat{Y}_U$ and $V^\dagger_{d_L} Y_D V_{d_R} = \hat{Y}_D$ (diagonal)

One thus obtains for the Quark part of $\mathcal{L}_{Mass}$

$$\mathcal{L}_{Mass} \supset -\frac{v}{\sqrt{2}} \left\{ \bar{u}_L \hat{Y}_U u_R + \bar{d}_L \hat{Y}_U d_R + h.c. \right\}$$

with the diagonal Yukawa matrices $\hat{Y}_u = \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix}$ and $\hat{Y}_d = \begin{pmatrix} y_s & & \\ & y_s & \\ & & y_b \end{pmatrix}$

One can identify the quark masses:

$$\mathbf{M}_u = \frac{v}{\sqrt{2}}\hat{Y}_U = \frac{v}{\sqrt{2}}V_{u_L}^{\dagger}Y_U V_{u_R} = diag\left(m_u, m_c, m_t\right)$$

$$\mathbf{M}_d = \frac{v}{\sqrt{2}}\hat{Y}_D = \frac{v}{\sqrt{2}}V_{d_L}^{\dagger}Y_D V_{d_R} = diag\left(m_d, m_s, m_b\right)$$

With $m_t \approx 173\,\text{GeV}$ and $v \approx 246\,\text{GeV}$ one finds:

$$y_t \approx 1 \gg y_u, y_d, y_s, y_c, y_b$$

The fact that the Yukawa couplings are so different is not understood, often referred as the flavor hierarchy problem (mass spectrum of the fermions).

3. Quark mixing in charged current interaction

As U_L and D_L have been independently transformed, there is a non-vanishing effect for the charged current terms in $\mathcal{L}_{Gauge}$ where the U_L and D_L enter both.

After electroweak symmetry breaking the charged current terms for the quarks are:

$$\mathcal{L}_{Gauge}^{CC} \supset \frac{g}{\sqrt{2}}\left\{\bar{U}_L\gamma_\mu W^{+\mu}D_L + \bar{D}_L\gamma_\mu W^{-\mu}U_L\right\} \qquad W^{\pm\mu} = \frac{1}{\sqrt{2}}\left(W_1^\mu \mp iW_2^\mu\right)$$

In the basis of the mass eigenstates u_L and d_L one obtains:

$$\mathcal{L}_{Gauge}^{CC} \supset \frac{g}{\sqrt{2}}\left\{\bar{u}_L \underbrace{V_{u_L}^\dagger V_{d_L}}_{V_{CKM}} \gamma_\mu W^{+\mu} d_L + \bar{d}_L \underbrace{V_{d_L}^\dagger V_{u_L}}_{V_{CKM}^\dagger} \gamma_\mu W^{-\mu} u_L\right\}$$

Remarks:
Neutral current terms are not affected as there are only $U_{L,R}$ or $D_{L,R}$ terms entering.
These terms are flavor diagonal: At tree-level there are no FCNC terms in the SM.

The mixing matrix V_{CKM} is called Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

By construction V_{CKM} is unitary:

$$V_{CKM}^{\dagger} V_{CKM} = 1$$

The states $d'_L = V_{CKM} d_L$ are often called weak eigenstates.

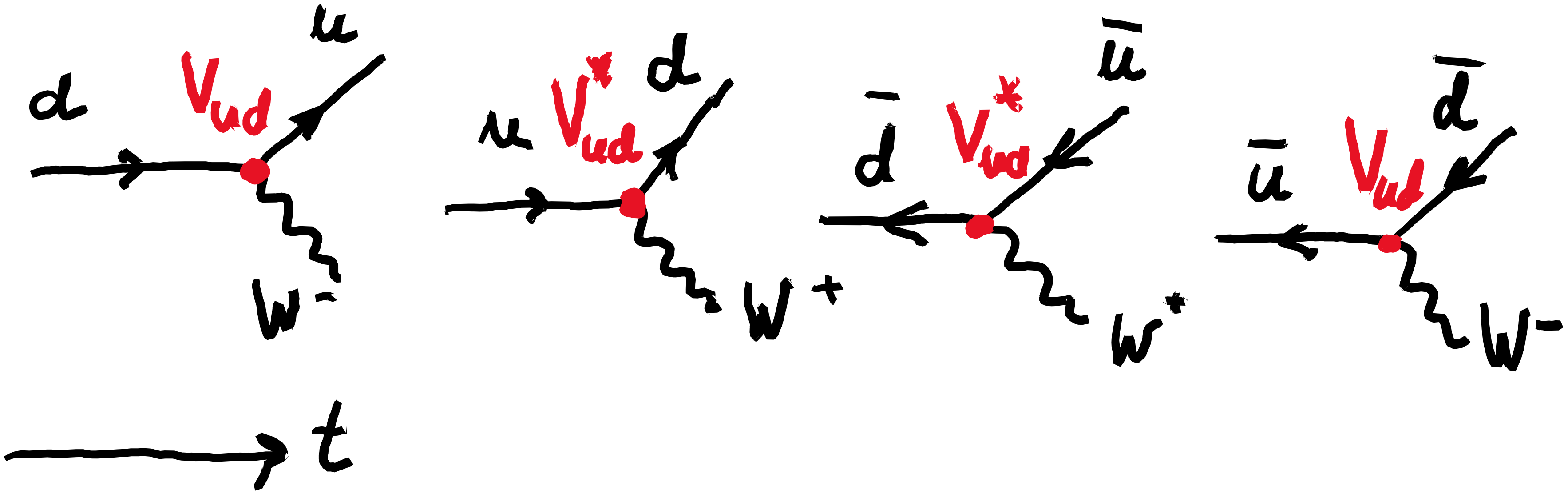
Number of free parameters:
In general, a 3×3 complex matrix has 18 parameters, the unitary condition reduces this to 9 free parameters.

As the quark phases are unobservable a rephasing is possible: $q_L \rightarrow e^{i\phi_q} q_L$
This allows to "rotate away" 5 phases (of the 9 free parameters) related to the phase differences:

$$V_{\alpha\beta} \rightarrow \exp\left(i\left[\phi_\beta - \phi_\alpha\right]\right) V_{\alpha\beta}$$

One overall phase and 3 other parameters ("rotation angles") are left.
⇒ The 3×3 CKM-matrix therefor has 4 independent parameters, and has complex elements with an imaginary part.

Pictorial representation of the quark charged-current interaction:



Remark on the mixing matrix:

In theories with N quark generations the N×N matrix has $\left(N-1\right)^2$ free parameters of which there are $\frac{1}{2}\left(N-1\right)\left(N-2\right)$ phases.

N=2: There is only one real parameter (angle).

Quark flavor sector:
Described by 6 masses + 4 CKM-parameters

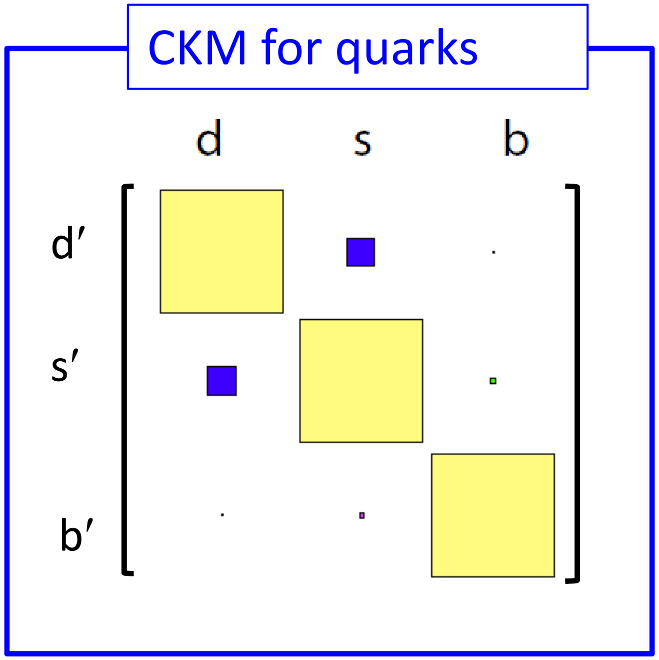
Lepton flavor sector:
Massless neutrinos: 3 masses (Standard Model)

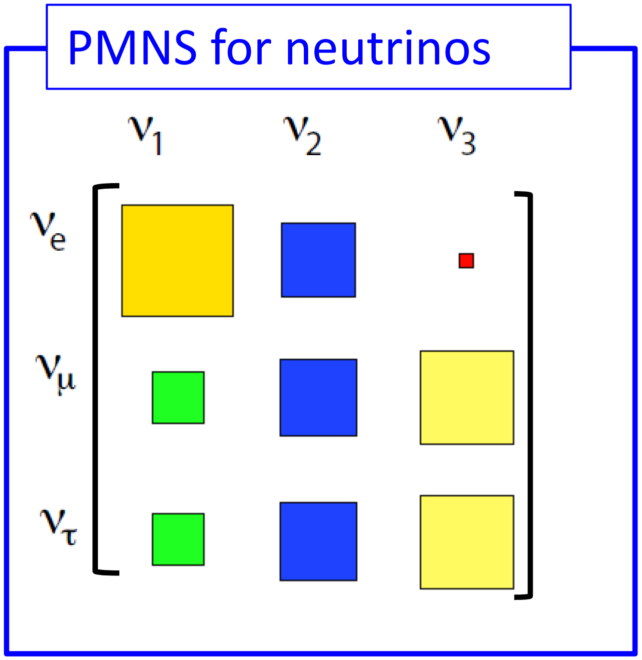
Massive neutrinos: 6 masses + 4 mixing-parameters (PMNS matrix)
(minimum set if neutrinos are Dirac particles)

So far, masses and mixing parameters are experimental inputs to theory. There is no deeper understanding of the mass hierarchy and the mixing parameters.

Relation between masses and mixing in the lepton and quark sector?

Mixing:


CKM for quarks


PMNS for neutrinos

Pontecorvo–Maki–Nakagawa–Sakata

Area of the squares is a measure of the magnitude of V_{ij}